

COMBINATIONS AND PERMUTATIONS: FUNDAMENTAL COUNTING PRINCIPLE AND COMBINATIONS*

Free High School Science Texts Project

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1 The Fundamental Counting Principle

The use of lists, tables and tree diagrams is only feasible for events with a few outcomes. When the number of outcomes grows, it is not practical to list the different possibilities and the fundamental counting principle is used.

The **fundamental counting principle** describes how to determine the total number of outcomes of a series of events.

Suppose that two experiments take place. The first experiment has n_1 possible outcomes, and the second has n_2 possible outcomes. Therefore, the first experiment, followed by the second experiment, will have a total of $n_1 \times n_2$ possible outcomes. This idea can be generalised to m experiments as the total number of outcomes for m experiments is:

$$n_1 \times n_2 \times n_3 \times \dots \times n_m = \prod_{i=1}^m n_i \quad (1)$$

\prod is the multiplication equivalent of \sum .

Note: the order in which the experiments are done does not affect the total number of possible outcomes.

Exercise 1: Lunch Special

(Solution on p. 4.)

A take-away has a 4-piece lunch special which consists of a sandwich, soup, dessert and drink for R25.00. They offer the following choices for :

Sandwich: chicken mayonnaise, cheese and tomato, tuna, and ham and lettuce

Soup: tomato, chicken noodle, vegetable

Dessert: ice-cream, piece of cake

Drink: tea, coffee, coke, Fanta and Sprite.

How many possible meals are there?

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2 Combinations

The fundamental counting principle describes how to calculate the total number of outcomes when multiple independent events are performed together.

A more complex problem is determining how many combinations there are of selecting a group of objects from a set. Mathematically, a *combination* is defined as an un-ordered collection of unique elements, or more formally, a subset of a set. For example, suppose you have fifty-two playing cards, and select five cards. The five cards would form a combination and would be a subset of the set of 52 cards.

In a set, the order of the elements in the set does not matter. These are represented usually with curly braces. For example $\{2, 4, 6\}$ is a subset of the set $\{1, 2, 3, 4, 5, 6\}$. Since the order of the elements does not matter, only the specific elements are of interest. Therefore,

$$\{2, 4, 6\} = \{6, 4, 2\} \quad (2)$$

and $\{1, 1, 1\}$ is the same as $\{1\}$ because in a set the elements don't usually appear more than once.

So in summary we can say the following: Given S , the set of all possible unique elements, a combination is a subset of the elements of S . The order of the elements in a combination is not important (two lists with the same elements in different orders are considered to be the same combination). Also, the elements cannot be repeated in a combination (every element appears once).

2.1 Counting Combinations

Calculating the number of ways that certain patterns can be formed is the beginning of *combinatorics*, the study of combinations. Let S be a set with n objects. Combinations of r objects from this set S are subsets of S having r elements each (where the order of listing the elements does not distinguish two subsets).

2.1.1 Combination without Repetition

When the order does not matter, but each object can be chosen only once, the number of combinations is:

$$\frac{n!}{r!(n-r)!} = \binom{n}{r} \quad (3)$$

where n is the number of objects from which you can choose and r is the number to be chosen.

For example, if you have 10 numbers and wish to choose 5 you would have $10! / (5!(10-5)!) = 252$ ways to choose.

For example how many possible 5 card hands are there in a deck of cards with 52 cards?

$52! / (5!(52-5)!) = 2\,598\,960$ combinations

2.1.2 Combination with Repetition

When the order does not matter and an object can be chosen more than once, then the number of combinations is:

$$\frac{(n+r-1)!}{r!(n-1)!} = \binom{n+r-1}{r} = \binom{n+r-1}{n-1} \quad (4)$$

where n is the number of objects from which you can choose and r is the number to be chosen.

For example, if you have ten types of donuts to choose from and you want three donuts there are $(10+3-1)! / 3!(10-1)! = 220$ ways to choose.

Note that in this video permutations are mentioned, you will cover permutations in the next section.

Khan academy video on probability - 1

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/bCxMhncR7PU&rel=0&hl=en_US&feature=player_embedded&version=3>

Figure 1

2.2 Combinatorics and Probability

Combinatorics is quite useful in the computation of probabilities of events, as it can be used to determine exactly how many outcomes are possible in a given experiment.

Exercise 2: Probability

(Solution on p. 4.)

At a school, learners each play 2 sports. They can choose from netball, basketball, soccer, athletics, swimming, or tennis. What is the probability that a learner plays soccer and either netball, basketball or tennis?

Solutions to Exercises in this Module

Solution to Exercise (p. 1)

Step 1. There are 4 parts: sandwich, soup, dessert and drink.

Step 2.

Meal component	Sandwich	Soup	Dessert	Drink
Number of choices	4	3	2	5

Table 1

Step 3.

$$4 \times 3 \times 2 \times 5 = 120 \quad (5)$$

So there are 120 possible meals.

Solution to Exercise (p. 3)

Step 1. We count the events: soccer and netball, soccer and basketball, soccer and tennis. This gives three choices.

Step 2. There are 6 sports to choose from and we choose 2 sports. There are $\binom{6}{2} = 6! / (2!(6-2)!) = 15$ choices.

Step 3. The probability is the number of events we are counting, divided by the total number of choices.
Probability = $\frac{3}{15} = \frac{1}{5} = 0,2$