Average Gradient: Parabolic and other functions*

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Free High School Science Texts Project

1 Average Gradient: Parabolic and other functions

1.1 Investigation : Average Gradient - Parabolic Function

Fill in the table by calculating the average gradient over the indicated intervals for the function $f(x) = 2x - 2$:

	x_1	x_2	y_1	y_2	$\frac{y_2 - y_1}{x_2 - x_1}$
A-B					
B-C					
C-D					
D-E					
E-F					
F-G					

Table 1

What do you notice about the average gradient over each interval? What can you say about the average gradients between A and D compared to the average gradients between D and G?

Figure 1

The average gradient of a parabolic function depends on the interval and is the gradient of a straight line that passes through the points on the interval.

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For example, in Figure 2 the various points have been joined by straight-lines. The average gradients between the joined points are then the gradients of the straight lines that pass through the points.

Figure 2: The average gradient between two points on a curve is the gradient of the straight line that passes through the points.

1.2 Method: Average Gradient

Given the equation of a curve and two points (x_1, x_2):

1. Write the equation of the curve in the form $y = ...$
2. Calculate y_1 by substituting x_1 into the equation for the curve.
3. Calculate y_2 by substituting x_2 into the equation for the curve.
4. Calculate the average gradient using:

$$\frac{y_2 - y_1}{x_2 - x_1} \tag{1}$$

Exercise 1: Average Gradient ***(Solution on p. 5.)***

Find the average gradient of the curve $y = 5x^2 - 4$ between the points $x = -3$ and $x = 3$

2 Average gradient for other functions

We can extend the concept of average gradient to any function. The average gradient for any function also depends on the interval chosen and is the gradient of a straight line that passes through the two points. So we can use the formula that we found for the average gradient of parabolic functions and apply it to any function. We will consider the average gradient of just two functions here: exponential functions and hyperbolic functions.

2.1 Average gradient of exponential functions

For example, if we were asked to find the average gradient of the function $\mathrm{g}(\mathrm{x}) = 3.2^x + 2$ between the points $(-4; 2, 2)$ and $(-0, 6; 4)$. This is shown in Figure 3.

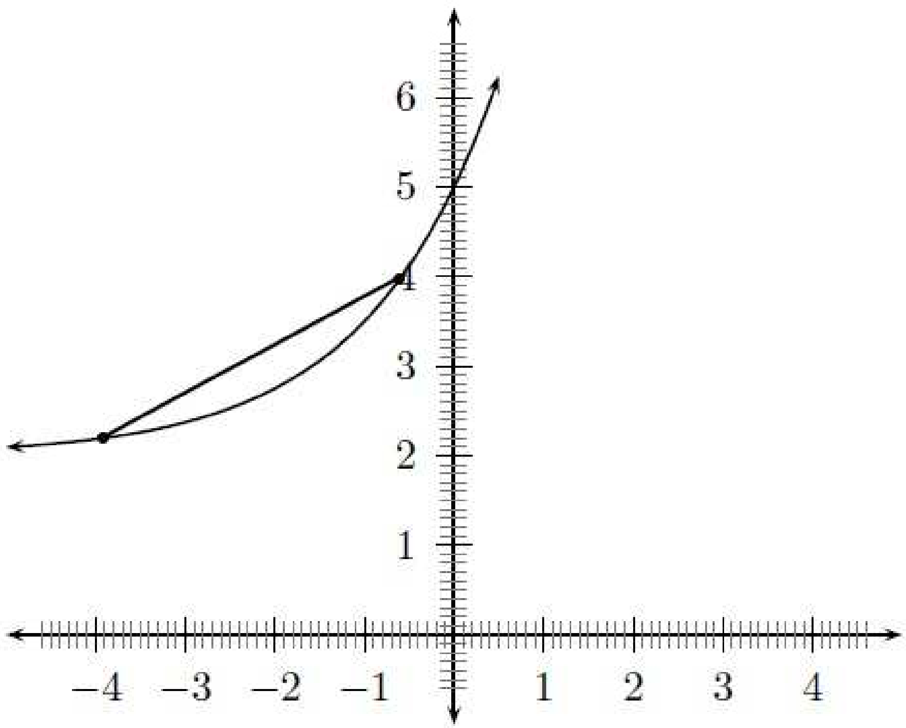

Figure 3: The average gradient for an exponential function.

Using the formula we find:

$$\begin{array}{rcl} \frac{y_2-y_1}{x_2-x_1} & = & \frac{4-2,2}{(-0,6)-(-4)} \\ & = & \frac{1,8}{-0,6+4} \\ & = & \frac{1,8}{5,2} \\ & = & 0,35 \end{array} \qquad (2)$$

2.2 Average gradient of hyperbolic functions

For example, if we were asked to find the average gradient of the function $g(x) = \frac{2}{x} + 2$ between the points $(-4;-2,5)$ and $(0,5;6)$ and $(-4;2,2)$ and $(-0,6;4)$. This is shown in Figure 4.

Figure 4: The average gradient for a hyperbolic function.

For the first point we would get:

$$\begin{array}{rcl} \frac{y_2-y_1}{x_2-x_1} & = & \frac{(-2,5)-1}{(-4)-0,5} \\ & = & \frac{-3,5}{-4,5} \\ & = & 0,78 \end{array} \qquad (3)$$

Similarly for the second points we would find that the average gradient is: $0,53$

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

Step 1. Label the points as follows:

$$x_1 = -3 \tag{4}$$

$$x_2 = 3 \tag{5}$$

to make it easier to calculate the gradient.

Step 2. We use the equation for the curve to calculate the y-value at x_1 and x_2.

$$\begin{array}{rcl} y_1 & = & 5x_1^2 - 4 \\ & = & 5(-3)^2 - 4 \\ & = & 5\,(9) - 4 \\ & = & 41 \end{array} \tag{6}$$

$$\begin{array}{rcl} y_2 & = & 5x_2^2 - 4 \\ & = & 5(3)^2 - 4 \\ & = & 5\,(9) - 4 \\ & = & 41 \end{array} \tag{7}$$

Step 3.

$$\begin{array}{rcl} \frac{y_2 - y_1}{x_2 - x_1} & = & \frac{41-41}{3-(-3)} \\ & = & \frac{0}{3+3} \\ & = & \frac{0}{6} \\ & = & 0 \end{array} \tag{8}$$

Step 4. The average gradient between $x = -3$ and $x = 3$ on the curve $y = 5x^2 - 4$ is 0.