

EQUATIONS AND INEQUALITIES: MATHEMATICAL MODELS*

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Free High School Science Texts Project

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1 Equations and inequalities: Mathematical models

1.1 Introduction

Tom and Jane are friends. Tom picked up Jane's Physics test paper, but will not tell Jane what her marks are. He knows that Jane hates maths so he decided to tease her. Tom says: 'I have 2 marks more than you do and the sum of both our marks is equal to 14. How much did we get?'

Let's help Jane find out what her marks are. We have two unknowns, Tom's mark (which we shall call t) and Jane's mark (which we shall call j). Tom has 2 more marks than Jane. Therefore,

$$t = j + 2 \tag{1}$$

Also, both marks add up to 14. Therefore,

$$t + j = 14 \tag{2}$$

The two equations make up a set of linear (because the highest power is one) simultaneous equations, which we know how to solve! Substitute for t in the second equation to get:

$$\begin{aligned} t + j &= 14 \\ j + 2 + j &= 14 \\ 2j + 2 &= 14 \\ 2(j + 1) &= 14 \\ j + 1 &= 7 \\ j &= 7 - 1 \\ &= 6 \end{aligned} \tag{3}$$

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Then,

$$\begin{aligned} t &= j + 2 \\ &= 6 + 2 \\ &= 8 \end{aligned} \tag{4}$$

So, we see that Tom scored 8 on his test and Jane scored 6.

This problem is an example of a simple *mathematical model*. We took a problem and we were able to write a set of equations that represented the problem mathematically. The solution of the equations then gave the solution to the problem.

1.2 Problem Solving Strategy

The purpose of this section is to teach you the skills that you need to be able to take a problem and formulate it mathematically in order to solve it. The general steps to follow are:

1. Read ALL of the question !
2. Find out what is requested.
3. Use a variable(s) to denote the unknown quantity/quantities that has/have been requested e.g., x .
4. Rewrite the information given in terms of the variable(s). That is, translate the words into algebraic expressions.
5. Set up an equation or set of equations (i.e. a mathematical sentence or model) to solve the required variable.
6. Solve the equation algebraically to find the result.

1.3 Application of Mathematical Modelling

Exercise 1: Mathematical Modelling: Two variables (Solution on p. 4.)

Three rulers and two pens have a total cost of R 21,00. One ruler and one pen have a total cost of R 8,00. How much does a ruler cost on its own and how much does a pen cost on its own?

Exercise 2: Mathematical Modelling: One variable (Solution on p. 4.)

A fruit shake costs R2,00 more than a chocolate milkshake. If three fruit shakes and 5 chocolate milkshakes cost R78,00, determine the individual prices.

1.3.1 Mathematical Models

1. Stephen has 1 l of a mixture containing 69% of salt. How much water must Stephen add to make the mixture 50% salt? Write your answer as a fraction of a litre. [Click here for the solution](#)¹
2. The diagonal of a rectangle is 25 cm more than its width. The length of the rectangle is 17 cm more than its width. What are the dimensions of the rectangle? [Click here for the solution](#)²
3. The sum of 27 and 12 is 73 more than an unknown number. Find the unknown number. [Click here for the solution](#)³
4. The two smaller angles in a right-angled triangle are in the ratio of 1:2. What are the sizes of the two angles? [Click here for the solution](#)⁴

¹<http://www.fhsst.org/lcy>

²<http://www.fhsst.org/lcV>

³<http://www.fhsst.org/lcp>

⁴<http://www.fhsst.org/lcw>

5. George owns a bakery that specialises in wedding cakes. For each wedding cake, it costs George R150 for ingredients, R50 for overhead, and R5 for advertising. George's wedding cakes cost R400 each. As a percentage of George's costs, how much profit does he make for each cake sold? Click here for the solution⁵
6. If 4 times a number is increased by 7, the result is 15 less than the square of the number. Find the numbers that satisfy this statement, by formulating an equation and then solving it. Click here for the solution⁶
7. The length of a rectangle is 2 cm more than the width of the rectangle. The perimeter of the rectangle is 20 cm. Find the length and the width of the rectangle. Click here for the solution⁷

⁵<http://www.fhsst.org/lcd>

⁶<http://www.fhsst.org/lcf>

⁷<http://www.fhsst.org/lcv>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

Step 1. Let the cost of one ruler be x rand and the cost of one pen be y rand.

Step 2.

$$\begin{aligned} 3x + 2y &= 21 \\ x + y &= 8 \end{aligned} \tag{5}$$

Step 3. First solve the second equation for y :

$$y = 8 - x \tag{6}$$

and substitute the result into the first equation:

$$\begin{aligned} 3x + 2(8 - x) &= 21 \\ 3x + 16 - 2x &= 21 \\ x &= 5 \end{aligned} \tag{7}$$

therefore

$$\begin{aligned} y &= 8 - 5 \\ y &= 3 \end{aligned} \tag{8}$$

Step 4. One ruler costs R 5,00 and one pen costs R 3,00.

Solution to Exercise (p. 2)

Step 1. Let the price of a chocolate milkshake be x and the price of a fruitshake be y .

| | Price | number | Total |
|-----------|-------|--------|-------|
| Fruit | y | 3 | $3y$ |
| Chocolate | x | 5 | $5x$ |

Table 1

Step 2.

$$3y + 5x = 78 \tag{9}$$

$$y = x + 2$$

Step 3.

$$\begin{aligned} 3(x + 2) + 5x &= 78 \\ 3x + 6 + 5x &= 78 \\ 8x &= 72 \\ x &= 9 \\ y &= x + 2 \\ &= 9 + 2 \\ &= 11 \end{aligned} \tag{10}$$

Step 4. One chocolate milkshake costs R 9,00 and one Fruitshake costs R 11,00