Probability: Part 2*

Free High School Science Texts Project

1 Relative Frequency *vs.* Probability

There are two approaches to determining the probability associated with any particular event of a random experiment:

1. determining the total number of possible outcomes and calculating the probability of each outcome using the definition of probability
2. performing the experiment and calculating the relative frequency of each outcome

Relative frequency is defined as the number of times an event happens in a statistical experiment divided by the number of trials conducted.

It takes a very large number of trials before the relative frequency of obtaining a head on a toss of a coin approaches the probability of obtaining a head on a toss of a coin (in fact, if the probability of an event occurring is something other than 1, i.e. it is always true, or 0, i.e. it is never true, then probability is only absolutely accurate when an infinite number of trials is conducted). For example, the data in Table 1 represent the outcomes of repeating 100 trials of a statistical experiment 100 times, i.e. tossing a coin 100 times.

H	T	T	H	H	T	H	H	H	H
H	H	H	H	T	H	H	T	T	T
T	T	H	T	T	H	T	H	T	H
H	H	T	T	H	T	T	H	T	T
T	H	H	H	T	T	H	T	T	H
H	T	T	T	T	H	T	T	H	H
T	T	H	T	T	H	T	T	H	T
H	T	T	H	T	T	T	T	H	T
T	H	T	T	H	H	H	T	H	T
T	T	T	H	H	T	T	T	H	T

Table 1: Results of 100 tosses of a fair coin. H means that the coin landed heads-up and T means that the coin landed tails-up.

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The following two worked examples show that the relative frequency of an event is not necessarily equal to the probability of the same event. Relative frequency should therefore be seen as an approximation to probability.

Exercise 1: Relative Frequency and Probability ***(Solution on p. 10.)***

Determine the relative frequencies associated with each outcome of the statistical experiment detailed in Table 1.

Exercise 2: Probability ***(Solution on p. 10.)***

Determine the probability associated with an evenly weighted coin landing on either of its faces.

2 Project Idea

Perform an experiment to show that as the number of trials increases, the relative frequency approaches the probability of a coin toss. Perform 10, 20, 50, 100, 200 trials of tossing a coin.

3 Interpretation of Probability Values - (not in CAPS, included for completeness)

The probability of an event is generally represented as a real number between 0 and 1, inclusive. An impossible event has a probability of exactly 0, and a certain event has a probability of 1, but the converses are not always true: probability 0 events are not always impossible, nor probability 1 events certain. There is a rather subtle distinction between "certain" and "probability 1".

For example, we can say that the sun will always rise in the east. This is a certain event, the sun will not suddenly rise in the north. But if we looked at the event of Penny Heyns winning a swimming race against your maths teacher, then this event is almost certain, since there is a very small chance that your teacher could win the race.

Most probabilities that occur in practice are numbers between 0 and 1, indicating the event's position on the continuum between impossibility and certainty. The closer an event's probability is to 1, the more likely it is to occur.

For example, if two mutually exclusive events are assumed equally probable, such as a flipped or spun coin landing heads-up or tails-up, we can express the probability of each event as "1 in 2", or, equivalently, "50%" or "1/2".

Probabilities are equivalently expressed as odds, which is the ratio of the probability of one event to the probability of all other events. The odds of heads-up, for the tossed/spun coin, are (1/2)/(1 - 1/2), which is equal to 1/1. This is expressed as "1 to 1 odds" and often written "1:1".

Odds a:b for some event are equivalent to probability a/(a+b). For example, 1:1 odds are equivalent to probability 1/2, and 3:2 odds are equivalent to probability 3/5.

4 Summary

- The term *random experiment* or *statistical experiment* is used to describe any repeatable process, the results of which are analyzed in some way.
- An outcome of an experiment is a single result of that experiment.
- The sample space of an experiment is the complete set of possible outcomes of the experiment.
- An event is any set of outcomes of an experiment.
- A Venn diagram can be used to show the relationship between the possible outcomes of a random experiment and the sample space. Venn diagrams can also be used to indicate the union and intersection between events in a sample space.
- When all outcomes are equally likely, they have an equal chance of happening. $P(E) = n(E)/n(S)$ gives us the probability of an equally likely outcome happening.
- *Relative frequency* is defined as the number of times an event happens in a statistical experiment divided by the number of trials conducted.

- The following results apply to probabilities, for the sample space S and two events A and B, within S.

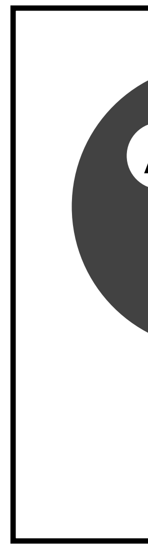
$$P\left(S\right)=1 \quad (1)$$

$$P\left(A\cap B\right)=P\left(A\right)\times P\left(B\right) \quad (2)$$

$$P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right) \quad (3)$$

- Mutually exclusive events are events, which cannot be true at the same time.
- $P\left(A'\right)=1-P\left(A\right)$ is the probability that A will not occur. This is known as a complementary event.

We can summarize some of the key concepts in this chapter in the following table:

Term	Meaning	Representation	Venn diagram
Union	Everything in A and B	$A\cup B$	

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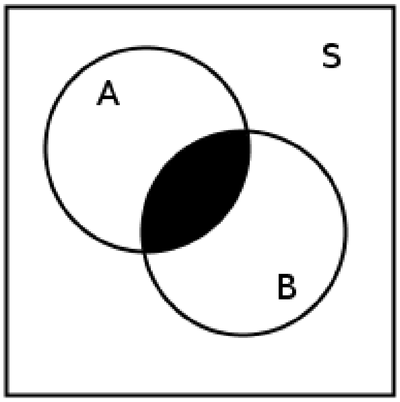
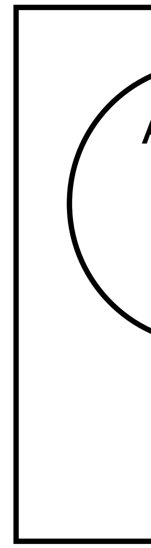
Intersection	Everything in A or B	$A \cap B$	

Complement	Everything that is not in A	A_c	

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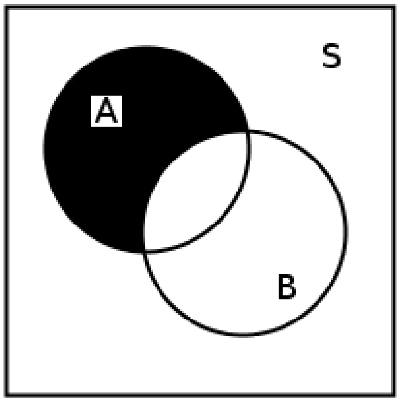
Only one	All that is only in A	$A - B$	

Table 2

5 End of Chapter Exercises

1. A group of 45 children were asked if they eat Frosties and/or Strawberry Pops. 31 eat both and 6 eat only Frosties. What is the probability that a child chosen at random will eat only Strawberry Pops? Click here for the solution.[1]
2. In a group of 42 pupils, all but 3 had a packet of chips or a Fanta or both. If 23 had a packet of chips and 7 of these also had a Fanta, what is the probability that one pupil chosen at random has:

   a. Both chips and Fanta
   b. has only Fanta?

   Click here for the solution.[2]
3. Use a Venn diagram to work out the following probabilities from a die being rolled:

   a. A multiple of 5 and an odd number
   b. a number that is neither a multiple of 5 nor an odd number
   c. a number which is not a multiple of 5, but is odd.

   Click here for the solution.[3]
4. A packet has yellow and pink sweets. The probability of taking out a pink sweet is 7/12.

   a. What is the probability of taking out a yellow sweet
   b. If 44 if the sweets are yellow, how many sweets are pink?

   Click here for the solution.[4]
5. In a car park with 300 cars, there are 190 Opels. What is the probability that the first car to leave the car park is:

   a. an Opel
   b. not an Opel

   Click here for the solution.[5]
6. Tamara has 18 loose socks in a drawer. Eight of these are orange and two are pink. Calculate the probability that the first sock taken out at random is:

   a. Orange
   b. not orange
   c. pink
   d. not pink
   e. orange or pink
   f. not orange or pink

   Click here for the solution.[6]
7. A plate contains 9 shortbread cookies, 4 ginger biscuits, 11 chocolate chip cookies and 18 Jambos. If a biscuit is selected at random, what is the probability that:

   a. it is either a ginger biscuit of a Jambo?
   b. it is NOT a shortbread cookie.

   Click here for the solution.[7]

[1]http://www.fhsst.org/lqh
[2]http://www.fhsst.org/llq
[3]http://www.fhsst.org/lll
[4]http://www.fhsst.org/lli
[5]http://www.fhsst.org/ll3
[6]http://www.fhsst.org/llO
[7]http://www.fhsst.org/llc

8. 280 tickets were sold at a raffle. Ingrid bought 15 tickets. What is the probability that Ingrid:
    a. Wins the prize
    b. Does not win the prize?

    Click here for the solution.[8]
9. The children in a nursery school were classified by hair and eye colour. 44 had red hair and not brown eyes, 14 had brown eyes and red hair, 5 had brown eyes but not red hair and 40 did not have brown eyes or red hair.
    a. How many children were in the school
    b. What is the probility that a child chosen at random has:
        1. Brown eyes
        2. Red hair
    c. A child with brown eyes is chosen randomly. What is the probability that this child will have red hair

    Click here for the solution.[9]
10. A jar has purple, blue and black sweets in it. The probability that a sweet, chosen at random, will be purple is 1/7 and the probability that it will be black is 3/5.
    a. If I choose a sweet at random what is the probability that it will be:
        i. purple or blue
        ii. Black
        iii. purple
    b. If there are 70 sweets in the jar how many purple ones are there?
    c. 1/4 if the purple sweets in b) have streaks on them and rest do not. How many purple sweets have streaks?

    Click here for the solution.[10]
11. For each of the following, draw a Venn diagram to represent the situation and find an example to illustrate the situation.
    a. A sample space in which there are two events that are not mutually exclusive
    b. A sample space in which there are two events that are complementary.

    Click here for the solution.[11]
12. Use a Venn diagram to prove that the probability of either event A or B occuring is given by: (A and B are not exclusive) P(A or B) = P(A) + P(B) - P(A and B)
    Click here for the solution.[12]
13. All the clubs are taken out of a pack of cards. The remaining cards are then shuffled and one card chosen. After being chosen, the card is replaced before the next card is chosen.
    a. What is the sample space?
    b. Find a set to represent the event, P, of drawing a picture card.
    c. Find a set for the event, N, of drawing a numbered card.
    d. Represent the above events in a Venn diagram
    e. What description of the sets P and N is suitable? (Hint: Find any elements of P in N and N in P.)

    Click here for the solution.[13]

[8] http://www.fhsst.org/llx
[9] http://www.fhsst.org/lla
[10] http://www.fhsst.org/llC
[11] http://www.fhsst.org/ll1
[12] http://www.fhsst.org/llr
[13] http://www.fhsst.org/llY

14. Thuli has a bag containing five orange, three purple and seven pink blocks. The bag is shaken and a block is withdrawn. The colour of the block is noted and the block is replaced.
    a. What is the sample space for this experiment?
    b. What is the set describing the event of drawing a pink block, P?
    c. Write down a set, O or B, to represent the event of drawing either a orange or a purple block.
    d. Draw a Venn diagram to show the above information.

    Click here for the solution.[14]

[14] http://www.fhsst.org/llg

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

Step 1. There are two unique outcomes: H and T.

Step 2.

Outcome	Frequency
H	44
T	56

Table 3

Step 3. The statistical experiment of tossing the coin was performed 100 times. Therefore, there were 100 trials, in total.

Step 4.

$$\begin{aligned} \text{Probability of H} &= \frac{\text{frequency of outcome}}{\text{number of trials}} \\ &= \frac{44}{100} \\ &= 0,44 \\ \text{Relative Frequency of T} &= \frac{\text{frequency of outcome}}{\text{number of trials}} \\ &= \frac{56}{100} \\ &= 0,56 \end{aligned} \tag{4}$$

The relative frequency of the coin landing heads-up is 0,44 and the relative frequency of the coin landing tails-up is 0,56.

Solution to Exercise (p. 2)

Step 1. There are two unique outcomes: H and T.

Step 2. There are two possible outcomes.

Step 3.

$$\begin{aligned} \text{Relative Frequency of H} &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{1}{2} \\ &= 0,5 \\ \text{Relative Frequency of T} &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{1}{2} \\ &= 0,5 \end{aligned} \tag{5}$$

The probability of an evenly weighted coin landing on either face is $0,5$.