Products and factors: Introduction and recap*

Free High School Science Texts Project

1 Introduction

In this chapter you will learn how to work with algebraic expressions. You will recap some of the work on factorisation and multiplying out expressions that you learnt in earlier grades. This work will then be extended upon for Grade 10.

2 Recap of Earlier Work

The following should be familiar. Examples are given as reminders.

2.1 Parts of an Expression

Mathematical expressions are just like sentences and their parts have special names. You should be familiar with the following names used to describe the parts of a mathematical expression.

$$\begin{aligned} a \cdot x^k + b \cdot x + c^m = 0 \\ d \cdot y^p + e \cdot y + f \leq 0 \end{aligned} \tag{1}$$

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Name	Examples (separated by commas)
term	$a \cdot x^k$,$b \cdot x$, c^m, $d \cdot y^p$, $e \cdot y$, f
expression	$a \cdot x^k + b \cdot x + c^m$, $d \cdot y^p + e \cdot y + f$
coefficient	a, b, d, e
exponent (or index)	k, p
base	x, y, c
constant	a, b, c, d, e, f
variable	x, y
equation	$a \cdot x^k + b \cdot x + c^m = 0$
inequality	$d \cdot y^p + e \cdot y + f \leq 0$
binomial	expression with two terms
trinomial	expression with three terms

Table 1

2.2 Product of Two Binomials

A *binomial* is a mathematical expression with two terms, e.g. $(ax + b)$ and $(cx + d)$. If these two binomials are multiplied, the following is the result:

$$\begin{aligned}(a \cdot x + b)(c \cdot x + d) &= (ax)(c \cdot x + d) + b(c \cdot x + d) \\ &= (ax)(cx) + (ax)d + b(cx) + b \cdot d \\ &= ax^2 + x(ad + bc) + bd\end{aligned} \quad (2)$$

Exercise 1: Product of two binomials ***(Solution on p. 5.)***

Find the product of $(3x - 2)(5x + 8)$

The product of two identical binomials is known as the *square of the binomial* and is written as:

$$(ax + b)^2 = a^2x^2 + 2abx + b^2 \quad (3)$$

If the two terms are $ax + b$ and $ax - b$ then their product is:

$$(ax + b)(ax - b) = a^2x^2 - b^2 \quad (4)$$

This is known as the *difference of two squares*.

2.3 Factorisation

Factorisation is the opposite of expanding brackets. For example expanding brackets would require $2(x + 1)$ to be written as $2x + 2$. Factorisation would be to start with $2x + 2$ and to end up with $2(x + 1)$. In previous grades, you factorised based on common factors and on difference of squares.

2.3.1 Common Factors

Factorising based on common factors relies on there being common factors between your terms. For example, $2x - 6x^2$ can be factorised as follows:

$$2x - 6x^2 = 2x\,(1 - 3x) \tag{5}$$

2.3.1.1 Investigation : Common Factors

Find the highest common factors of the following pairs of terms:

(a) $6y; 18x$	(b) $12mn; 8n$	(c) $3st; 4su$	(d) $18kl; 9kp$	(e) $abc; ac$
(f) $2xy; 4xyz$	(g) $3uv; 6u$	(h) $9xy; 15xz$	(i) $24xyz; 16yz$	(j) $3m; 45n$

Table 2

2.3.2 Difference of Two Squares

We have seen that:

$$(ax + b)\,(ax - b) = a^2x^2 - b^2 \tag{6}$$

Since (6) is an equation, both sides are always equal. This means that an expression of the form:

$$a^2x^2 - b^2 \tag{7}$$

can be factorised to

$$(ax + b)\,(ax - b) \tag{8}$$

Therefore,

$$a^2x^2 - b^2 = (ax + b)\,(ax - b) \tag{9}$$

For example, $x^2 - 16$ can be written as $\left(x^2 - 4^2\right)$ which is a difference of two squares. Therefore, the factors of $x^2 - 16$ are $(x - 4)$ and $(x + 4)$.

Exercise 2: Factorisation ***(Solution on p. 5.)***

Factorise completely: $b^2y^5 - 3aby^3$

Exercise 3: Factorising binomials with a common bracket ***(Solution on p. 5.)***

Factorise completely: $3a\,(a - 4) - 7\,(a - 4)$

Exercise 4: Factorising using a switch around in brackets ***(Solution on p. 5.)***

Factorise $5\,(a - 2) - b\,(2 - a)$

2.3.2.1 Recap

1. Find the products of:

(a) $2y(y+4)$	(b) $(y+5)(y+2)$	(c) $(y+2)(2y+1)$
(d) $(y+8)(y+4)$	(e) $(2y+9)(3y+1)$	(f) $(3y-2)(y+6)$

Table 3

Click here for the solution[1]

2. Factorise:
   a. $2l+2w$
   b. $12x+32y$
   c. $6x^2+2x+10x^3$
   d. $2xy^2+xy^2z+3xy$
   e. $-2ab^2-4a^2b$

   Click here for the solution[2]

3. Factorise completely:

(a) $7a+4$	(b) $20a-10$	(c) $18ab-3bc$
(d) $12kj+18kq$	(e) $16k^2-4k$	(f) $3a^2+6a-18$
(g) $-6a-24$	(h) $-2ab-8a$	(i) $24kj-16k^2j$
(j) $-a^2b-b^2a$	(k) $12k^2j+24k^2j^2$	(l) $72b^2q-18b^3q^2$
(m) $4(y-3)+k(3-y)$	(n) $a(a-1)-5(a-1)$	(o) $bm(b+4)-6m(b+4)$
(p) $a^2(a+7)+a(a+7)$	(q) $3b(b-4)-7(4-b)$	(r) $a^2b^2c^2-1$

Table 4

Click here for the solution[3]

[1] http://www.fhsst.org/lxI
[2] http://www.fhsst.org/lqV
[3] http://www.fhsst.org/lqE

Solutions to Exercises in this Module

Solution to Exercise **(p. 2)**

Step 1.

$$\begin{aligned}(3x-2)(5x+8) &= (3x)(5x)+(3x)(8)+(-2)(5x)+(-2)(8)\\ &= 15x^2+24x-10x-16\\ &= 15x^2+14x-16\end{aligned} \quad (10)$$

Solution to Exercise **(p. 3)**

Step 1.

$$b^2y^5-3aby^3 = by^3\left(by^2-3a\right) \quad (11)$$

Solution to Exercise **(p. 3)**

Step 1. $(a-4)$ is the common factor

$$3a(a-4)-7(a-4) = (a-4)(3a-7) \quad (12)$$

Solution to Exercise **(p. 3)**

Step 1.

$$\begin{aligned}5(a-2)-b(2-a) &= 5(a-2)-[-b(a-2)]\\ &= 5(a-2)+b(a-2)\\ &= (a-2)(5+b)\end{aligned} \quad (13)$$