

PRODUCTS AND FACTORS: FRACTIONS*

Free High School Science Texts Project

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1 Simplification of Fractions

In some cases of simplifying an algebraic expression, the expression will be a fraction. For example,

$$\frac{x^2 + 3x}{x + 3} \quad (1)$$

has a quadratic in the numerator and a binomial in the denominator. You can apply the different factorisation methods to simplify the expression.

$$\begin{aligned} & \frac{x^2+3x}{x+3} \\ = & \frac{x(x+3)}{x+3} \\ = & x \quad \text{provided } x \neq -3 \end{aligned} \quad (2)$$

If x were 3 then the denominator, $x - 3$, would be 0 and the fraction undefined.

Exercise 1: Simplification of Fractions

(Solution on p. 5.)

Simplify: $\frac{2x-b+x-ab}{ax^2-abx}$

Exercise 2: Simplification of Fractions

(Solution on p. 5.)

Simplify: $\frac{x^2-x-2}{x^2-4} \div \frac{x^2+x}{x^2+2x}$

1.1 Simplification of Fractions

1. Simplify:

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(a) $\frac{3a}{15}$	(b) $\frac{2a+10}{4}$
(c) $\frac{5a+20}{a+4}$	(d) $\frac{a^2-4a}{a-4}$
(e) $\frac{3a^2-9a}{2a-6}$	(f) $\frac{9a+27}{9a+18}$
(g) $\frac{6ab+2a}{2b}$	(h) $\frac{16x^2y-8xy}{12x-6}$
(i) $\frac{4xyp-8xp}{12xy}$	(j) $\frac{3a+9}{14} \div \frac{7a+21}{a+3}$
(k) $\frac{a^2-5a}{2a+10} \div \frac{3a+15}{4a}$	(l) $\frac{3xp+4p}{8p} \div \frac{12p^2}{3x+4}$
(m) $\frac{16}{2xp+4x} \div \frac{6x^2+8x}{12}$	(n) $\frac{24a-8}{12} \div \frac{9a-3}{6}$
(o) $\frac{a^2+2a}{5} \div \frac{2a+4}{20}$	(p) $\frac{p^2+pq}{7p} \div \frac{8p+8q}{21q}$
(q) $\frac{5ab-15b}{4a-12} \div \frac{6b^2}{a+b}$	(r) $\frac{f^2a-fa^2}{f-a}$

Table 1

Click here for the solution¹

2. Simplify: $\frac{x^2-1}{3} \times \frac{1}{x-1} - \frac{1}{2}$

Click here for the solution²

2 Adding and subtracting fractions

Using the concepts learnt in simplification of fractions, we can now add and subtract simple fractions. To add or subtract fractions we note that we can only add or subtract fractions that have the same denominator. So we must first make all the denominators the same and then perform the addition or subtraction. This is called finding the lowest common denominator or multiple.

For example, if you wanted to add: $\frac{1}{2}$ and $\frac{3}{5}$ we would note that the lowest common denominator is 10. So we must multiply the first fraction by 5 and the second fraction by 2 to get both of these with the same denominator. Doing so gives: $\frac{5}{10}$ and $\frac{6}{10}$. Now we can add the fractions. Doing so, we get $\frac{11}{10}$.

Exercise 3

Simplify the following expression: $\frac{x-2}{x^2-4} + \frac{x^2}{x-2} - \frac{x^3+x-4}{x^2-4}$

(Solution on p. 5.)

3 Two interesting mathematical proofs

We can use the concepts learnt in this chapter to demonstrate two interesting mathematical proofs. The first proof states that $n^2 + n$ is even for all $n \in \mathbb{Z}$. The second proof states that $n^3 - n$ is divisible by 6 for all $n \in \mathbb{Z}$. Before we demonstrate that these two laws are true, we first need to note some other mathematical rules.

If we multiply an even number by an odd number, we get an even number. Similarly if we multiply an odd number by an even number we get an even number. Also, an even number multiplied by an even number is even and an odd number multiplied by an odd number is odd. This result is shown in the following table:

	Odd number	Even number
Odd number	Odd	Even
Even number	Even	Even

¹<http://www.fhsst.org/lit>

²<http://www.fhsst.org/lie>

Table 2

If we take three consecutive numbers and multiply them together, the resulting number is always divisible by three. This should be obvious since if we have any three consecutive numbers, one of them will be divisible by 3.

Now we are ready to demonstrate that $n^2 + n$ is even for all $n \in Z$. If we factorise this expression we get: $n(n + 1)$. If n is even, then $n + 1$ is odd. If n is odd, then $n + 1$ is even. Since we know that if we multiply an even number with an odd number or an odd number with an even number, we get an even number, we have demonstrated that $n^2 + n$ is always even. Try this for a few values of n and you should find that this is true.

To demonstrate that $n^3 - n$ is divisible by 6 for all $n \in Z$, we first note that the factors of 6 are 3 and 2. So if we show that $n^3 - n$ is divisible by both 3 and 2, then we have shown that it is also divisible by 6! If we factorise this expression we get: $n(n + 1)(n - 1)$. Now we note that we are multiplying three consecutive numbers together (we are taking n and then adding 1 or subtracting 1. This gives us the two numbers on either side of n .) For example, if $n = 4$, then $n + 1 = 5$ and $n - 1 = 3$. But we know that when we multiply three consecutive numbers together, the resulting number is always divisible by 3. So we have demonstrated that $n^3 - n$ is always divisible by 3. To demonstrate that it is also divisible by 2, we can also show that it is even. We have shown that $n^2 + n$ is always even. So now we recall what we said about multiplying even and odd numbers. Since one number is always even and the other can be either even or odd, the result of multiplying these numbers together is always even. And so we have demonstrated that $n^3 - n$ is divisible by 6 for all $n \in Z$.

4 Summary

- A binomial is a mathematical expression with two terms. The product of two identical binomials is known as the square of the binomial. The difference of two squares is when we multiply $(ax + b)(ax - b)$
- Factorising is the opposite of expanding the brackets. You can use common factors or the difference of two squares to help you factorise expressions.
- The distributive law $((A + B)(C + D + E) = A(C + D + E) + B(C + D + E))$ helps us to multiply a binomial and a trinomial.
- The sum of cubes is: $(x + y)(x^2 - xy + y^2) = x^3 + y^3$ and the difference of cubes is: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- To factorise a quadratic we find the two binomials that were multiplied together to give the quadratic.
- We can also factorise a quadratic by grouping. This is where we find a common factor in the quadratic and take it out and then see what is left over.
- We can simplify fractions by using the methods we have learnt to factorise expressions.
- Fractions can be added or subtracted. To do this the denominators of each fraction must be the same.

5 End of Chapter Exercises

1. Factorise:

- $a^2 - 9$
- $m^2 - 36$
- $9b^2 - 81$
- $16b^6 - 25a^2$
- $m^2 - (1/9)$
- $5 - 5a^2b^6$
- $16ba^4 - 81b$
- $a^2 - 10a + 25$
- $16b^2 + 56b + 49$

- j. $2a^2 - 12ab + 18b^2$
- k. $-4b^2 - 144b^8 + 48b^5$

Click here for the solution³

2. Factorise completely:

- a. $(16 - x^4)$
- b. $7x^2 - 14x + 7xy - 14y$
- c. $y^2 - 7y - 30$
- d. $1 - x - x^2 + x^3$
- e. $-3(1 - p^2) + p + 1$

Click here for the solution⁴

3. Simplify the following:

- a. $(a - 2)^2 - a(a + 4)$
- b. $(5a - 4b)(25a^2 + 20ab + 16b^2)$
- c. $(2m - 3)(4m^2 + 9)(2m + 3)$
- d. $(a + 2b - c)(a + 2b + c)$

Click here for the solution⁵

4. Simplify the following:

- a. $\frac{p^2 - q^2}{p} \div \frac{p + q}{p^2 - pq}$
- b. $\frac{2}{x} + \frac{x}{2} - \frac{2x}{3}$

Click here for the solution⁶

5. Show that $(2x - 1)^2 - (x - 3)^2$ can be simplified to $(x + 2)(3x - 4)$

Click here for the solution⁷

6. What must be added to $x^2 - x + 4$ to make it equal to $(x + 2)^2$

Click here for the solution⁸

³<http://www.fhsst.org/liM>

⁴<http://www.fhsst.org/lTY>

⁵<http://www.fhsst.org/lTg>

⁶<http://www.fhsst.org/lT4>

⁷<http://www.fhsst.org/lib>

⁸<http://www.fhsst.org/liT>

Solutions to Exercises in this Module

Solution to Exercise (p. 1)

Step 1. Use *grouping* for numerator and *common factor* for denominator in this example.

$$\begin{aligned}
 &= \frac{(ax-ab)+(x-b)}{ax^2-abx} \\
 &= \frac{a(x-b)+(x-b)}{ax(x-b)} \\
 &= \frac{(x-b)(a+1)}{ax(x-b)}
 \end{aligned} \tag{3}$$

Step 2. The simplified answer is:

$$= \frac{a+1}{ax} \tag{4}$$

Solution to Exercise (p. 1)

Step 1.

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \div \frac{x(x+1)}{x(x+2)} \tag{5}$$

Step 2.

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \times \frac{x(x+2)}{x(x+1)} \tag{6}$$

Step 3. The simplified answer is

$$= 1 \tag{7}$$

Solution to Exercise (p. 2)

Step 1.

$$\frac{x-2}{(x+2)(x-2)} + \frac{x^2}{x-2} - \frac{x^3+x-4}{(x+2)(x-2)} \tag{8}$$

Step 2. We make all the denominators the same so that we can add or subtract the fractions. The lowest common denominator is $(x-2)(x+2)$.

$$\frac{x-2}{(x+2)(x-2)} + \frac{(x^2)(x+2)}{(x+2)(x-2)} - \frac{x^3+x-4}{(x+2)(x-2)} \tag{9}$$

Step 3. Since the fractions all have the same denominator we can write them all as one fraction with the appropriate operator

$$\frac{x-2+(x^2)(x+2)-x^3+x-4}{(x+2)(x-2)} \tag{10}$$

Step 4.

$$\frac{2x^2+2x-6}{(x+2)(x-2)} \tag{11}$$

Step 5.

$$\frac{2(x^2+x-3)}{(x+2)(x-2)} \tag{12}$$