Flow Rate and Its Relation to Velocity*

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Abstract

- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

Flow rate$Q$ is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in Figure 1. In symbols, this can be written as

$$Q = \frac{V}{t}, \tag{1}$$

where V is the volume and t is the elapsed time.

The SI unit for flow rate is m^3/s, but a number of other units for Q are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a **liter** (L) is 1/1000 of a cubic meter or 1000 cubic centimeters (10^{-3} m^3 or 10^3 cm^3). In this text we shall use whatever metric units are most convenient for a given situation.

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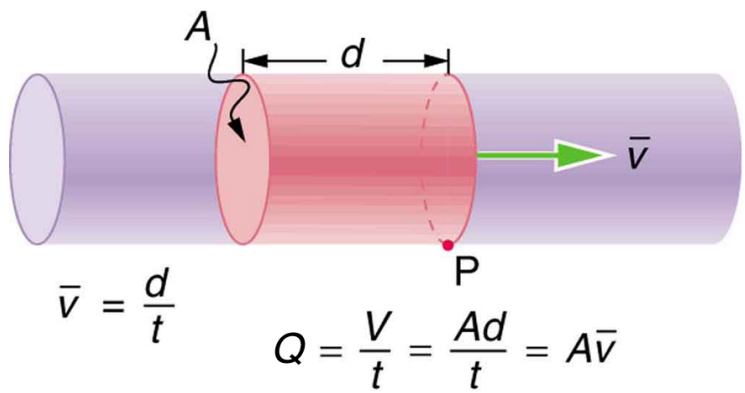

Figure 1: Flow rate is the volume of fluid per unit time flowing past a point through the area A. Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t. The volume of the cylinder is Ad and the average velocity is $\overline{v} = d/t$ so that the flow rate is $Q = \mathrm{Ad}/t = A\overline{v}$.

Example 1: Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Strategy

Time and flow rate Q are given, and so the volume V can be calculated from the definition of flow rate.

Solution

Solving $Q = V/t$ for volume gives

$$V = \mathrm{Qt}. \tag{2}$$

Substituting known values yields

$$\begin{aligned} V &= \left(\tfrac{5.00\ \mathrm{L}}{1\ \mathrm{min}}\right)(75\ \mathrm{y})\left(\tfrac{1\ \mathrm{m}^3}{10^3\ \mathrm{L}}\right)\left(5.26 \times 10^5\ \tfrac{\mathrm{min}}{\mathrm{y}}\right) \\ &= 2.0 \times 10^5\ \mathrm{m}^3. \end{aligned} \tag{3}$$

Discussion

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think about the flow rate of a river. The greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate Q and velocity $\overline{v}$ is

$$Q = A\overline{v}, \tag{4}$$

where A is the cross-sectional area and $\overline{v}$ is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity

(hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. Figure 1 illustrates how this relationship is obtained. The shaded cylinder has a volume

$$V = \mathrm{Ad}, \tag{5}$$

which flows past the point P in a time t. Dividing both sides of this relationship by t gives

$$\frac{V}{t} = \frac{\mathrm{Ad}}{t}. \tag{6}$$

We note that $Q = V/t$ and the average speed is $\overline{v} = d/t$. Thus the equation becomes $Q = A\overline{v}$.

Figure 2 shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

$$\left.\begin{array}{r} Q_1 = Q_2 \\ A_1\overline{v}_1 = A_2\overline{v}_2 \end{array}\right\}. \tag{7}$$

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.

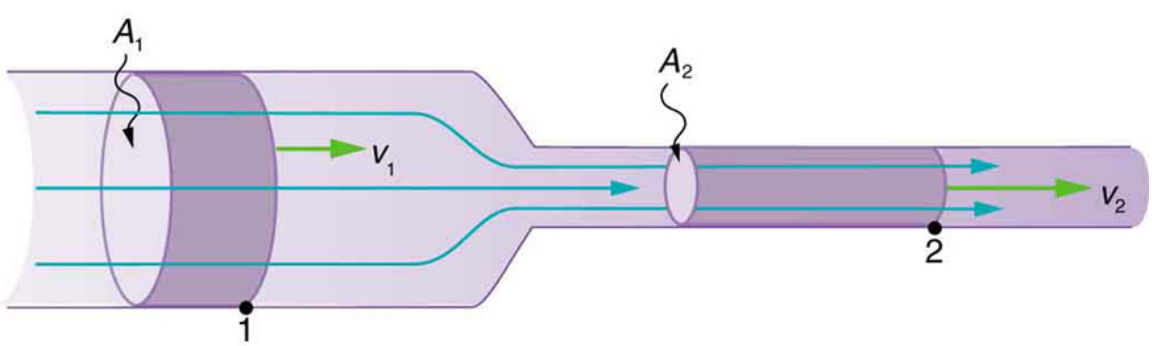

Figure 2: When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.

Example 2: Calculating Fluid Speed: Speed Increases When a Tube Narrows

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Strategy

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

Solution for (a)

First, we solve $Q = A\overline{v}$ for v_1 and note that the cross-sectional area is $A = \pi \mathrm{r}^2$, yielding

$$\overline{v}_1 = \frac{Q}{A_1} = \frac{Q}{\pi \mathrm{r}_1^2}. \tag{8}$$

Substituting known values and making appropriate unit conversions yields

$$\overline{v}_1 = \frac{(0.500\ \mathrm{L/s})\left(10^{-3}\ \mathrm{m^3/L}\right)^2}{\pi\left(9.00 \times 10^{-3}\ \mathrm{m}\right)} = 1.96\ \mathrm{m/s}. \tag{9}$$

Solution for (b)

We could repeat this calculation to find the speed in the nozzle $\overline{v}_2$, but we will use the equation of continuity to give a somewhat different insight. Using the equation which states

$$A_1\overline{v}_1 = A_2\overline{v}_2, \tag{10}$$

solving for $\overline{v}_2$ and substituting $\pi \mathrm{r}^2$ for the cross-sectional area yields

$$\overline{v}_2 = \frac{A_1}{A_2}\overline{v}_1 = \frac{\pi \mathrm{r}_1^2}{\pi \mathrm{r}_2^2}\overline{v}_1 = \frac{r_1{}^2}{r_2{}^2}\overline{v}_1. \tag{11}$$

Substituting known values,

$$\overline{v}_2 = \frac{(0.900\ \mathrm{cm})^2}{(0.250\ \mathrm{cm})}^2\ 1.96\ \mathrm{m/s} = 25.5\ \mathrm{m/s}. \tag{12}$$

Discussion

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

$$n_1 A_1 \overline{v}_1 = n_2 A_2 \overline{v}_2, \tag{13}$$

where n_1 and n_2 are the number of branches in each of the sections along the tube.

Example 3: Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known

as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0 μm, calculate the number of capillaries in the blood circulatory system.

Strategy

We can use $Q = A\overline{v}$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

Solution for (a)

The flow rate is given by $Q = A\overline{v}$ or $\overline{v} = \frac{Q}{\pi r^2}$ for a cylindrical vessel.

Substituting the known values (converted to units of meters and seconds) gives

$$\overline{v} = \frac{(5.0\ \mathrm{L/min})\left(10^{-3}\ \mathrm{m}^3/\mathrm{L}\right)(1\ \mathrm{min}/60\ \mathrm{s})}{\pi(0.010\ \mathrm{m})^2} = 0.27\ \mathrm{m/s}\ . \quad (14)$$

Solution for (b)

Using $n_1 A_1 \overline{v}_1 = n_2 A_2 \overline{v}_1$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives $n_2 = \frac{n_1 A_1 \overline{v}_1}{A_2 \overline{v}_2}$. Converting all quantities to units of meters and seconds and substituting into the equation above gives

$$n_2 = \frac{(1)(\pi)\left(10^{-3}\ \mathrm{m}\right)^2(0.27 m/s)}{(\pi)\left(4.0\times 10^{-6}\ \mathrm{m}\right)^2\left(0.33\times 10^{-3}\ \mathrm{m/s}\right)} = 5.0\times 10^{9}\ \text{capillaries}\ . \quad (15)$$

Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per mm^3, or about 200×10^6 per 1 kg of muscle. For 20 kg of muscle, this amounts to about 4×10^9 capillaries.

1 Section Summary

- Flow rate Q is defined to be the volume V flowing past a point in time t, or $Q = \frac{V}{t}$ where V is volume and t is time.
- The SI unit of volume is m^3.
- Another common unit is the liter (L), which is $10^{-3}\ \mathrm{m}^3$.
- Flow rate and velocity are related by $Q = A\overline{v}$ where A is the cross-sectional area of the flow and $\overline{v}$ is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is,

$$\left.\begin{aligned} Q_1 &= Q_2 \\ A_1\overline{v}_1 &= A_2\overline{v}_2 \\ n_1 A_1 \overline{v}_1 &= n_2 A_2 \overline{v}_2 \end{aligned}\right\}. \quad (16)$$

2 Conceptual Questions

Exercise 1
What is the difference between flow rate and fluid velocity? How are they related?

Exercise 2
Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)

Exercise 3
Identify some substances that are incompressible and some that are not.

3 Problems & Exercises

Exercise 4 ***(Solution on p. 9.)***
What is the average flow rate in cm^3/s of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?

Exercise 5
The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to cm^3/s. (b) What is this rate in m^3/s?

Exercise 6 ***(Solution on p. 9.)***
Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.

Exercise 7
Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.

Exercise 8 ***(Solution on p. 9.)***
The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see Figure 3). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?

Figure 3: The Huka Falls in Taupo, New Zealand, demonstrate flow rate. (credit: RaviGogna, Flickr)

Exercise 9

A major artery with a cross-sectional area of $1.00\ \text{cm}^2$ branches into 18 smaller arteries, each with an average cross-sectional area of $0.400\ \text{cm}^2$. By what factor is the average velocity of the blood reduced when it passes into these branches?

Exercise 10 ***(Solution on p. 9.)***

(a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is $10.0\ \text{cm}^2$, what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is $10.0\ \mu\text{m}$?

Exercise 11

The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about $8\ \mu\text{m}$. Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.

Exercise 12 ***(Solution on p. 9.)***

(a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at $5000\ \text{m}^3/\text{s}$, into it?

Exercise 13

The flow rate of blood through a 2.00×10^{-6}-m -radius capillary is $3.80 \times 10^9\ \text{cm}^3/\text{s}$. (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of $90.0\ \text{cm}^3/\text{s}$? (The large number obtained is an overestimate, but it is still reasonable.)

Exercise 14 ***(Solution on p. 9.)***

(a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

Exercise 15

The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.

Exercise 16 ***(Solution on p. 9.)***

Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?

Exercise 17

Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

Exercise 18 ***(Solution on p. 9.)***

Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in cm^3/s? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

Exercise 19

Unreasonable Results

A mountain stream is 10.0 m wide and averages 2.00 m in depth. During the spring runoff, the flow in the stream reaches 100,000 m^3/s. (a) What is the average velocity of the stream under these conditions? (b) What is unreasonable about this velocity? (c) What is unreasonable or inconsistent about the premises?

Solutions to Exercises in this Module

Solution to Exercise (p. 6)
$2.78\ \mathrm{cm}^3/\mathrm{s}$
Solution to Exercise (p. 6)
27 cm/s
Solution to Exercise (p. 6)
(a) 0.75 m/s
(b) 0.13 m/s
Solution to Exercise (p. 7)
(a) $40.0\ \mathrm{cm}^2$
(b) 5.09×10^7
Solution to Exercise (p. 7)
(a) 22 h
(b) 0.016 s
Solution to Exercise (p. 7)
(a) 12.6 m/s
(b) $0.0800\ \mathrm{m}^3/\mathrm{s}$
(c) No, independent of density.
Solution to Exercise (p. 8)
(a) 0.402 L/s
(b) 0.584 cm
Solution to Exercise (p. 8)
(a) $127\ \mathrm{cm}^3/\mathrm{s}$
(b) 0.891 cm

Glossary

Definition 1: flow rate
abbreviated Q, it is the volume V that flows past a particular point during a time t, or $Q = V/t$

Definition 2: liter
a unit of volume, equal to $10^{-3}\ \mathrm{m}^3$