Magnetic Force between Two Parallel Conductors*

OpenStax College

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Abstract

- Describe the effects of the magnetic force between two conductors.
- Calculate the force between two parallel conductors.

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to *define* the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance r can be found by applying what we have developed in preceding sections. Figure 1 shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance r is given to be

$$B_1 = \frac{\mu_0 I_1}{2\pi \mathrm{r}}. \tag{1}$$

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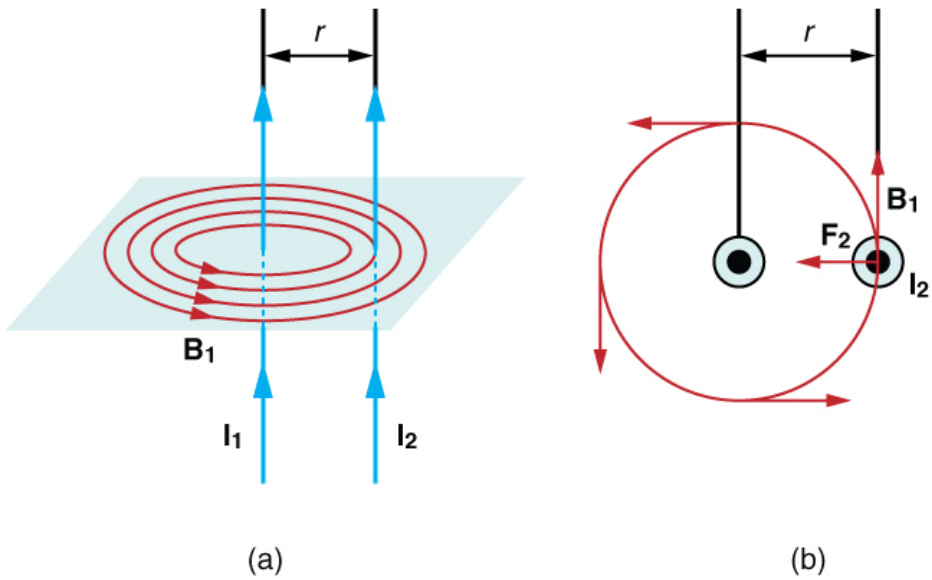

Figure 1: (a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for each wire. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

This field is uniform along wire 2 and perpendicular to it, and so the force F_2 it exerts on wire 2 is given by $F = \mathrm{IlB}\sin\theta$ with $\sin\theta = 1$:

$$F_2 = I_2 \mathrm{lB}_1. \tag{2}$$

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write F for the magnitude of F_2. (Note that $F_1 = -F_2$.) Since the wires are very long, it is convenient to think in terms of F/l, the force per unit length. Substituting the expression for B_1 into the last equation and rearranging terms gives

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi \mathrm{r}}. \tag{3}$$

F/l is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The *operational definition of the ampere* is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

$$\frac{F}{l} = \frac{\left(4\pi \times 10^{-7}\ T \cdot \mathrm{m/A}\right)(1A)^2}{(2\pi)(1m)} = 2 \times 10^{-7}\ \mathrm{N/m}. \tag{4}$$

Since μ_0 is exactly $4\pi \times 10^{-7}\ T \cdot \mathrm{m/A}$ by definition, and because $1\ \mathrm{T} = 1\ \mathrm{N}/(A \cdot m)$, the force per meter is exactly $2 \times 10^{-7}\ \mathrm{N/m}$. This is the basis of the operational definition of the ampere.

: The official definition of the ampere is:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly $2 \times 10^{-7}\ \mathrm{N/m}$ on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1C = 1A \cdot s$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

1 Section Summary

- The force between two parallel currents I_1 and I_2, separated by a distance r, has a magnitude per unit length given by
$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi \mathrm{r}}. \tag{5}$$
- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

2 Conceptual Questions

Exercise 1

Is the force attractive or repulsive between the hot and neutral lines hung from power poles? Why?

Exercise 2

If you have three parallel wires in the same plane, as in Figure 2, with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.

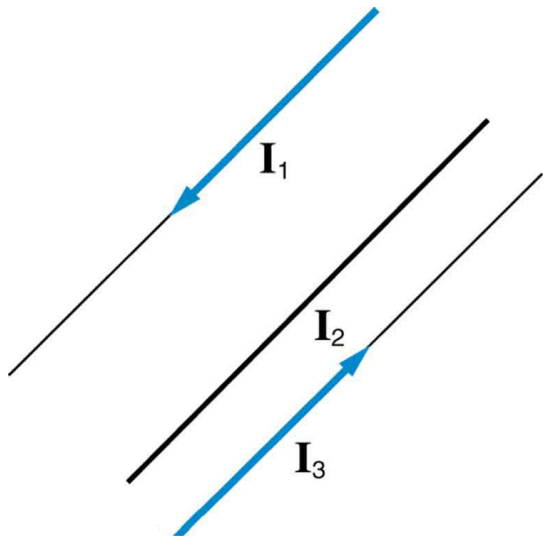

Figure 2: Three parallel coplanar wires with currents in the outer two in opposite directions.

Exercise 3

Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right hand rules.

Exercise 4

Use the right hand rules to show that the force between the two loops in Figure 3 is attractive if the currents are in the same direction and repulsive if they are in opposite directions. Is this consistent with like poles of the loops repelling and unlike poles of the loops attracting? Draw sketches to justify your answers.

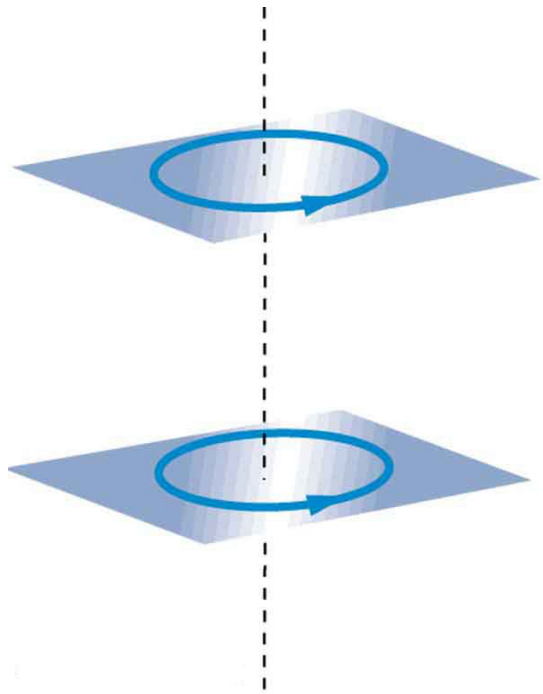

Figure 3: Two loops of wire carrying currents can exert forces and torques on one another.

Exercise 5
If one of the loops in Figure 3 is tilted slightly relative to the other and their currents are in the same direction, what are the directions of the torques they exert on each other? Does this imply that the poles of the bar magnet-like fields they create will line up with each other if the loops are allowed to rotate?

Exercise 6
Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?

3 Problems & Exercises

Exercise 7 ***(Solution on p. 8.)***
(a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires? (b) Discuss the practical consequences of this force, if any.

Exercise 8
The force per meter between the two wires of a jumper cable being used to start a stalled car is 0.225 N/m. (a) What is the current in the wires, given they are separated by 2.00 cm? (b) Is the force attractive or repulsive?

Exercise 9 ***(Solution on p. 8.)***
A 2.50-m segment of wire supplying current to the motor of a submerged submarine carries 1000

A and feels a 4.00-N repulsive force from a parallel wire 5.00 cm away. What is the direction and magnitude of the current in the other wire?

Exercise 10

The wire carrying 400 A to the motor of a commuter train feels an attractive force of 4.00×10^{-3} N/m due to a parallel wire carrying 5.00 A to a headlight. (a) How far apart are the wires? (b) Are the currents in the same direction?

Exercise 11 ***(Solution on p. 8.)***

An AC appliance cord has its hot and neutral wires separated by 3.00 mm and carries a 5.00-A current. (a) What is the average force per meter between the wires in the cord? (b) What is the maximum force per meter between the wires? (c) Are the forces attractive or repulsive? (d) Do appliance cords need any special design features to compensate for these forces?

Exercise 12

Figure 4 shows a long straight wire near a rectangular current loop. What is the direction and magnitude of the total force on the loop?

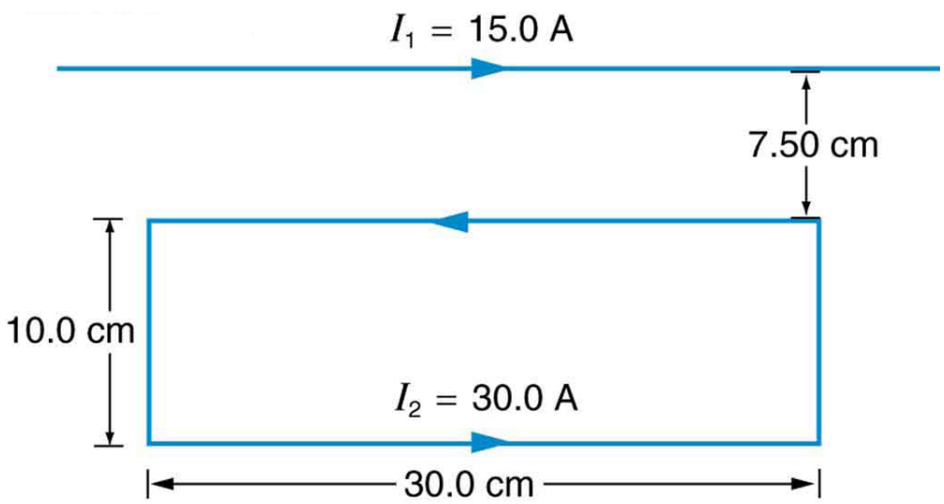

Figure 4

Exercise 13 ***(Solution on p. 8.)***

Find the direction and magnitude of the force that each wire experiences in Figure 5(a) by, using vector addition.

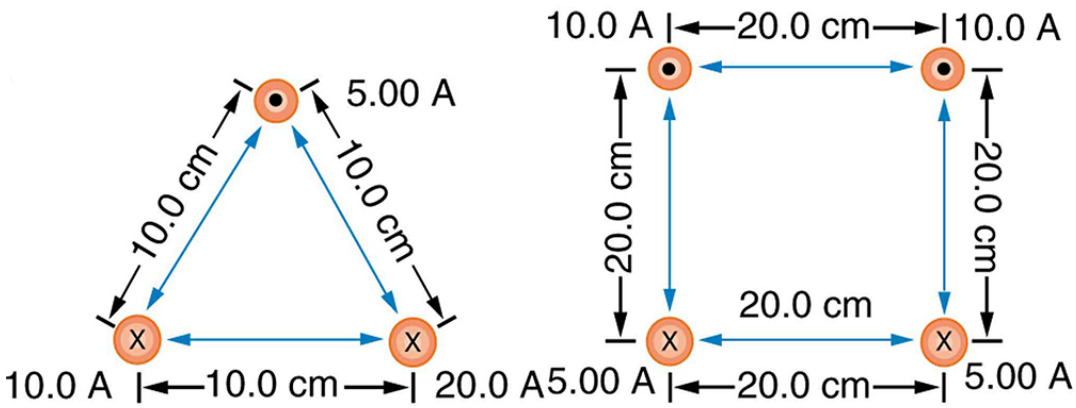

Figure 5

Exercise 14

Find the direction and magnitude of the force that each wire experiences in Figure 5(b), using vector addition.

Solutions to Exercises in this Module

Solution to Exercise (p. 5)
(a) 8.53 N, repulsive
(b) This is about 2 pounds of force for 50 m of wire, which is not very significant given that the wires are held at points more closely spaced than 50 m.

Solution to Exercise (p. 5)
400 A in the opposite direction

Solution to Exercise (p. 6)
(a) 1.67×10^{-3} N/m
(b) 3.33×10^{-3} N/m
(c) Repulsive
(d) No, these are very small forces

Solution to Exercise (p. 6)
(a) Top wire: 2.65×10^{-4} N/m s, 10.9 to left of up
(b) Lower left wire: 3.61×10^{-4} N/m, 13.9 down from right
(c) Lower right wire: 3.46×10^{-4} N/m, 30.0 down from left