

GAME 2302-0330 SCALE FACTORS, RATIOS, AND PROPORTIONS*

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Abstract

This module provides a brief tutorial on scale factors, ratios, and proportions.

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2 Preface

2.1 General

This module is part of a series of modules designed for teaching the physics component of *GAME2302 Mathematical Applications for Game Development* at Austin Community College in Austin, TX. (See *GAME 2302-0100: Introduction*¹ for the first module in the course along with a description of the course, course resources, homework assignments, etc.)

The module provides a brief tutorial on scale factors, ratios, and proportions.

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¹<http://cnx.org/content/m44983/latest>

2.2 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

2.2.1 Figures

- Figure 1 (p. 4) . Screen output for Listing #1.
- Figure 2 (p. 6) . Screen output for Listing #2.
- Figure 3 (p. 8) . Screen output for Listing #3.

2.2.2 Listings

- Listing 1 (p. 3) . Exercise on scale factors
- Listing 2 (p. 5) . Circumference is proportional to radius.
- Listing 3 (p. 7) . Area is proportional to radius squared.

3 General background information

Mathematical expressions are used in physics to describe relationships that are difficult to express in words. The expressions use algebraic symbols to represent quantities that consist of numbers and units.

Measurements are important

Conclusions that are drawn in physics and other sciences ranging from chemistry to the social sciences are often based on measurements such as length, width, weight, salinity, population, density, etc.

Each number in an equation often represents the results of a measurement, which is made in terms of a standard. The units indicate which standard was used to make the measurements.

Knowledge of units is critical

A number that is used to indicate the result of a measurement is of little value unless we know the units in which the measurement was made. For example, it isn't very useful to know that the length of an object is 125 unless we know whether the units are meters, centimeters, millimeters, or miles.

The Google calculator

Although I won't go into detail in this module, I will tell you that the Google calculator is very good at helping you to keep track of units. For example, if you enter the following expression in the Google search box,

$$3 \text{ ft} + 1 \text{ yd} + 36 \text{ inches}$$

The following result will be displayed immediately below the search box:

$$(3 \text{ ft}) + (1 \text{ yd}) + (36 \text{ inches}) = 2.7432 \text{ meters}$$

4 Discussion

We often express the relationship between two items using a scale factor.

4.1 Scale factors

For example, we might say that a colt doubled its weight in one year. This means that the colt's weight after one year was equal to the colt's weight at birth multiplied by a factor of two. The factor is the number by which a quantity is multiplied or divided when it is changed from one value to another.

4.2 Ratios

The factor is the ratio of the new value to the old value. Regarding the colt mentioned above, we might write:

$$\text{NewWeight} / \text{BirthWeight} = 2$$

where the slash character "/" indicates division.

Percentage increases

It is also common for us to talk about something increasing or decreasing by a given percentage value. For example, we might say that a colt increased its weight by 50-percent in one year. This is equivalent to saying:

$$\text{NewWeight} = \text{BirthWeight} * (1 + 50/100)$$

This assumes that when evaluating a mathematical expression:

- The asterisk character "*" indicates multiplication.
- Computations begin inside of the inner-most pair of matching parentheses and work outward from there.
- Multiplication and division are performed before addition and subtraction are performed.

This is typical behavior for computer programs and spreadsheets, but not necessarily for hand calculators.

Percentage decreases

In my dreams, I might say that I went on a diet and my weight decreased by 25-percent in one year. This would be equivalent to saying:

$$\text{NewWeight} = \text{OldWeight} * (1 - 25/100)$$

Exercise on scale factors

Write a script that has the following behavior. Given a chalk line that is 100 inches long, draw other chalk lines that are:

- A. Twice the length of the original line.
- B. Twenty-five percent of the length of the original line.
- C. Twenty-five percent greater than the length of the original line.
- D. Twenty-five percent less than the length of the original line.

My version of the script is shown in Listing 1 (p. 3) .

Listing 1: Exercise on scale factors

```
<!-- File JavaScript01.html -->
<html><body>
<script language="JavaScript1.3">

//Do the computations
var origLine = 100
var lineA = 2 * origLine
var lineB = (25/100) * origLine
var lineC = (1 + 25/100) * origLine
var lineD = (1 - 25/100) * origLine

//Display the results
document.write("Original line = "
    + origLine + "<br/>")
document.write("A = " + lineA + "<br/>")
document.write("B = " + lineB + "<br/>")
```

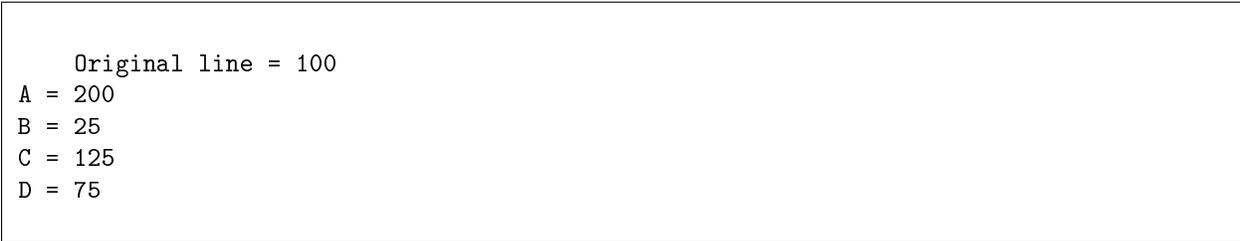
```
document.write("C = " + lineC + "<br/>")
document.write("D = " + lineD + "<br/>")

</script>
</body></html>
```

Screen output

When you copy the code from Listing 1 (p. 3) into an html file and open the file in your web browser, the output text shown in Figure 1 (p. 4) should appear in your browser window.

Screen output for Listing #1.



```
Original line = 100
A = 200
B = 25
C = 125
D = 75
```

Figure 1: Screen output for Listing #1.

Note that although they sound similar, specifications B and D above (p. 3) don't mean the same thing.

4.3 Proportions

We talk about increasing or changing a value by some factor because we can often simplify a problem by thinking in terms of proportions.

NOTE: A symbol for proportionality

Physics textbooks often use a character that doesn't appear on a QWERTY keyboard to indicate "is proportional to."

I will use a "\$" character for that purpose because:

- It does appear on a QWERTY keyboard.
- It isn't typically used in mathematical expressions unless American currency is involved.
- It is not a JavaScript operator.

For example, I will write $A \$ B$ to indicate that A is proportional to B.

When we say that A is proportional to B, or

$A \$ B$

we mean that if B increases by some factor, then A must increase by the same factor.

Circumference of a circle

Let's illustrate what we mean with a couple of examples. For the first example, we will consider the circumference of a circle. Hopefully, you know that the circumference of a circle is given by the expression:

$$C = 2 * \text{PI} * r$$

where:

- C is the circumference of the circle
- PI is the mathematical constant 3.14159...
- r is the radius of the circle

From this expression, we can conclude that

$$C \propto r$$

If we modify the radius...

If we double the radius, the circumference will also double. If we reduce the radius by 25-percent, the circumference will also be reduced by 25-percent. This is illustrated by the script in Listing 2 (p. 5) .

Listing 2: Circumference is proportional to radius.

```
<!-- File JavaScript02.html -->
<html><body>
<script language="JavaScript1.3">

var r = 10
var C = 2 * Math.PI * r
document.write("r =" + r +
               ", C = " + C + "<br/>")

//Multiply r by 2. Then display r and C
r = r * 2
C = 2 * Math.PI * r
document.write("r =" + r +
               ", C = " + C + "<br/>")

//Reduce r by 25%, Then display r and C
r = r * (1 - 25/100)
C = 2 * Math.PI * r
document.write("r =" + r +
               ", C = " + C + "<br/>")

</script>
</body></html>
```

Output from the script

When you open the script shown in Listing 2 (p. 5) in your browser, the text shown in Figure 2 (p. 6) should appear in your browser window.

Screen output for Listing #2.

```
r =10, C = 62.83185307179586
r =20, C = 125.66370614359172
r =15, C = 94.24777960769379
```

Figure 2: Screen output for Listing #2.

Figure 2 (p. 6) shows the value of the circumference for three different values for the radius. You should be able to confirm that the combination of the three lines of output text satisfy the proportionality rules stated earlier (p. 5) .

For example, you can confirm these results by entering the following three expressions in the Google search box and recording the results that appear immediately below the search box:

$2*\pi*10$

$2*\pi*20$

$2*\pi*15$

Area of a circle

Before I can discuss the area of a circle, I need to define a symbol that we can use to indicate exponentiation.

NOTE: A symbol for exponentiation

Physics textbooks typically use a superscript character to indicate that a value is raised to a power, such as the radius of a circle squared.

When I need to indicate that a value is raised to a power, I will use the "^" character, such as in the following text that indicates radius squared, or radius raised to the second power.

radius²

In those cases where the exponent is a fraction, or is negative, I will surround it with parentheses, such as in

radius^(1/2), and

distance⁽⁻²⁾

The first term indicates the square root of the radius. The second term indicates that distance is being raised to the -2 power.

I chose to use the "^" character to indicate exponentiation because it is used as the exponentiation operator in some programming languages, such as BASIC. The "^" character is also recognized by the Google calculator as an exponentiation operator.

Unfortunately, there is no exponentiation operator in JavaScript, so we will need a different approach to raise a value to a power in our JavaScript scripts. As you will see later, we will use the built-in

Math.pow method for that purpose.

An expression for the area of a circle

From your earlier coursework, you should know that the area of a circle is given by

$$A = \text{PI} * r^2$$

where

- A is the area.
- PI is the mathematical constant 3.14159...
- r is the radius of the circle.

Proportional to the square of the radius

From this, we can conclude that the area of a circle is not proportional to the radius. Instead, it is proportional to the square of the radius as in

$$A \propto r^2$$

If you change the radius...

If you change the value of the radius, the area changes in proportion to the square of the radius. If the radius doubles, the area increases by four. If the radius is decreased by 25-percent, the area decreases by more than 25-percent. This is illustrated by the script in Listing 3 (p. 7) .

Listing 3: Area is proportional to radius squared.

```
<!-- File JavaScript03.html -->
<html><body>
<script language="JavaScript1.3">

var r = 10
var A = Math.PI * Math.pow(r,2)
document.write("r =" + r +
               ", A = " + A + "<br/>")

//Multiply r by 2. Then display r and C
r = r * 2
var A = Math.PI * Math.pow(r,2)
document.write("r =" + r +
               ", A = " + A + "<br/>")

//Reduce r by 25%, Then display r and C
r = r * (1 - 25/100)
var A = Math.PI * Math.pow(r,2)
document.write("r =" + r +
               ", A = " + A + "<br/>")

//Compute and the display the cube root
// of a number.
var X = Math.pow(8,1/3)
document.write("Cube root of 8 = " + X)

</script>
</body></html>
```

The JavaScript Math.pow method

Listing 3 (p. 7) calls a built-in JavaScript method that I have not used before: **Math.pow** . This method is called to raise a value to a power. It requires two parameters. The first parameter is the value that is to be raised to a power and the second parameter is the power to which the value is to be raised.

The method returns the value raised to the power.

Fractional exponents

Although this topic is not directly related to the discussion on proportionality, as long as I am introducing the method named **Math.pow** , I will point out that it is legal for the exponent to be a fraction. The last little bit of code in Listing 3 (p. 7) raises the value 8 to the $1/3$ power. This actually computes the cube root of the value 8. As you should be able to confirm in your head, the cube root of 8 is 2, because two raised to the third power is 8.

Output from the script

When you open the script shown in Listing 3 (p. 7) in your browser, the text shown in Figure 3 (p. 8) should appear in your browser window.

Screen output for Listing #3.

```
r =10, A = 314.1592653589793
r =20, A = 1256.6370614359173
r =15, A = 706.8583470577034
Cube root of 8 = 2
```

Figure 3: Screen output for Listing #3.

An examination of the first three lines of text in Figure 3 (p. 8) should confirm that they satisfy the proportionality rules for the square of the radius described earlier (p. 7) .

The last line of text in Figure 3 (p. 8) confirms that the **Math.pow** method can be used to compute roots by specifying fractional exponents as the second parameter.

5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of .html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

6 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: GAME 2302-0330 Scale Factors, Ratios, and Proportions
- Revised: 10/12/12

- File: Game0330.htm

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