Moments And Vanishing Wavelet Moments

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Definition 1: $k^{th}$ order moments of $f(t)$

$$m[k] = \int t^k f(t) \, dt$$

Definition 2: $k^{th}$ order moments of $s(n)$

$$\mu[k] = \sum_{nn} n^k s[n]$$

Definition 3: Partial Moments

$$v(k, l) = \sum_{nn} (2n + l)^k s[2n + l]$$

Notation

- $m[k]$: scaling function moments
- $m_1[k]$: wavelet function moments
- $\mu[k]$: scaling filter moments
- $\mu_1[k]$: wavelet filter moments

Example 1
For a random variable $x$ with a pdf $p(x)$, its mean value is

First order moment of $p(x)$

$$E[x] = \int x p(x) \, dx \quad (1)$$

If $x$ is zero-mean, its variance is

Second order moment of $p(x)$

$$\text{Var}(x) = \int x^2 p(x) \, dx \quad (2)$$
Moments may have special meaning in physics and mechanics field.

**Example 2**

The centroid of an object with density \( \rho (r) \) is defined as

\[
C = \int r \rho (r) \, dr
\]

Its moment of inertia is defined as

\[
I = \int r^2 \rho (r) \, dr
\]

**NOTE:** The wavelet transform of a function \( f (t) \in V_j \) can be written as

\[
f (t) = \sum_{kk} c_{j0} (k) \phi_{j0,k} (t) + \sum_{kk} \sum_{j=j0}^{J-1} d_j (k) \psi_{j,k} (t)
\]

where \( c_{j0} = \langle f (t), \phi_{j0,k} (t) \rangle \) and \( d_j (k) = \langle f (t), \psi_{j,k} (t) \rangle \) are scaling coefficients and wavelet coefficients, respectively. If the signal \( f (t) \) is in polynomial form, the coefficients \( c \)'s and \( d \)'s are linear combination of different order moments.

If we can make the moments of wavelet function to be zero up to a certain order \( K - 1 \), for any polynomial with order lower than \( K \), all its wavelet coefficients will be zero, or be **vanishing**. The signal will then fall entirely into the scaling function space, or we can say the scaling function has power to represent polynomials of degree up to \( K - 1 \). Such a wavelet system is said to have **vanishing wavelet moments**.

Making a wavelet system to have vanishing wavelet moments up to order \( K - 1 \) is equivalent to putting regularity on its scaling filter, which is known as "\( K \)-regularity" of scaling filter.