TRIGONOMETRY: THE TRIG FUNCTIONS
(Grade 10) [NCS]*

Free High School Science Texts Project

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1 Definition of the Trigonometric Functions

We are familiar with a function of the form \( f(x) \) where \( f \) is the function and \( x \) is the argument. Examples are:

\[
\begin{align*}
f(x) & = 2^x \quad \text{(exponential function)} \\
g(x) & = x + 2 \quad \text{(linear function)} \\
h(x) & = 2x^2 \quad \text{(parabolic function)} 
\end{align*}
\] (1)

The basis of trigonometry are the **trigonometric functions**. There are three basic trigonometric functions:

1. sine
2. cosine
3. tangent

These are abbreviated to:

1. \( \sin \)
2. \( \cos \)
3. \( \tan \)

These functions are defined from a **right-angled triangle**, a triangle where one internal angle is 90 °. Consider a right-angled triangle.

*Image not finished*

**Figure 1**
In the right-angled triangle, we refer to the lengths of the three sides according to how they are placed in relation to the angle $\theta$. The side opposite to the right angle is labelled the hypotenuse, the side opposite $\theta$ is labelled opposite, the side next to $\theta$ is labelled adjacent. Note that the choice of non-90 degree internal angle is arbitrary. You can choose either internal angle and then define the adjacent and opposite sides accordingly. However, the hypotenuse remains the same regardless of which internal angle you are referring to.

We define the trigonometric functions, also known as trigonometric identities, as:

\[
\begin{align*}
\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\tan \theta &= \frac{\text{opposite}}{\text{adjacent}}
\end{align*}
\]

These functions relate the lengths of the sides of a right-angled triangle to its interior angles.

One way of remembering the definitions is to use the following mnemonic that is perhaps easier to remember:

| Silly Old Hens | Sin = \frac{\text{Opposite}}{\text{Hypotenuse}} |
| Cackle And Howl | Cos = \frac{\text{Adjacent}}{\text{Hypotenuse}} |
| Till Old Age | Tan = \frac{\text{Opposite}}{\text{Adjacent}} |

Table 1

You may also hear people saying Soh Cah Toa. This is just another way to remember the trig functions.

**Tip:** The definitions of opposite, adjacent and hypotenuse are only applicable when you are working with right-angled triangles! Always check to make sure your triangle has a right-angle before you use them, otherwise you will get the wrong answer. We will find ways of using our knowledge of right-angled triangles to deal with the trigonometry of non right-angled triangles in Grade 11.

1.1 Investigation: Definitions of Trigonometric Functions

1. In each of the following triangles, state whether $a$, $b$ and $c$ are the hypotenuse, opposite or adjacent sides of the triangle with respect to the marked angle.

2. Complete each of the following, the first has been done for you
\[ a \ \sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CB}{AC} \]
\[ b \ \cos A = \]
\[ c \ \tan A = \]
\[ d \ \sin C = \]
\[ e \ \cos C = \]
\[ f \ \tan C = \]

3. Complete each of the following without a calculator:

Figure 4

\[
\begin{align*}
\sin 60 &= \\
\cos 30 &= \\
\tan 60 &= 
\end{align*}
\]

Figure 5

\[
\begin{align*}
\sin 45 &= \\
\cos 45 &= \\
\tan 45 &= 
\end{align*}
\]

For most angles \( \theta \), it is very difficult to calculate the values of \( \sin \theta \), \( \cos \theta \) and \( \tan \theta \). One usually needs to use a calculator to do so. However, we saw in the above Activity that we could work these values out for some special angles. Some of these angles are listed in the table below, along with the values of the trigonometric functions at these angles. Remember that the lengths of the sides of a right angled triangle must obey Pythagoras’ theorem. The square of the hypothenuse (side opposite the 90 degree angle) equals the sum of the squares of the two other sides.
These values are useful when asked to solve a problem involving trig functions without using a calculator.

Exercise 1: Finding Lengths
Find the length of x in the following triangle.

![Image not finished](Figure 6)

Exercise 2: Finding Angles
Find the value of θ in the following triangle.

![Image not finished](Figure 6)

The following videos provide a summary of what you have learnt so far.

Trigonometry - 1
This media object is a Flash object. Please view or download it at <http://www.youtube.com/v/F2189Wpi0y8&rel=0>

Khan academy video on trigonometry - 2
This media object is a Flash object. Please view or download it at <http://www.youtube.com/v/QS4r_mps-rY&rel=0>
1.2 Finding Lengths
Find the length of the sides marked with letters. Give answers correct to 2 decimal places.

Click here for the solution.\(^\text{1}\)

\(^\text{1}\)http://www.fhsst.org/lc1

http://cnx.org/content/m39900/1.1/
Solutions to Exercises in this Module

Solution to Exercise (p. 4)

Step 1. In this case you have an angle (50°), the opposite side and the hypotenuse. So you should use \( \sin \)

\[
\sin 50^\circ = \frac{x}{100}
\]  

(7)

Step 2.

\[
\Rightarrow x = 100 \times \sin 50^\circ
\]  

(8)

Step 3. Use the sin button on your calculator

\[
\Rightarrow x = 76.6m
\]  

(9)

Solution to Exercise (p. 4)

Step 1. In this case you have the opposite side and the hypotenuse to the angle \( \theta \). So you should use \( \tan \)

\[
\tan \theta = \frac{50}{100}
\]  

(10)

Step 2.

\[
\Rightarrow \tan \theta = 0.5
\]  

(11)

Step 3. Since you are finding the angle, use \( \tan^{-1} \) on your calculator

Don't forget to set your calculator to 'deg' mode!

\[
\Rightarrow \theta = 26.6^\circ
\]  

(12)