STRAIN GAGES AND FORCE MEASUREMENT*

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1 Introduction

In this lab, you will learn about strain gages and the Wheatstone bridge circuit. You will see how they can be used for strain and force measurement. You will modify an existing program to measure the dynamic characteristics of a second-order system.

2 Teaching Objectives

- Gain practical experience with resistance strain-measurement techniques.
- Learn about the Wheatstone bridge and how it is used in strain measurement.
- Use a beam instrumented with strain gages as a force measurement device.
- Use strain gages to measure the natural frequency and damping in a beam.
- Design a force transducer for measuring thrust from a model rocket motor.

3 Preparatory Reading:

Figliola and Beasley
Strain Measurement: pp. 425-446

4 Procedure

4.1 Part 1: Strain Gages and the Wheatstone Bridge

The metal foil strain gages used in this lab are resistors with a nominal (unstrained) resistance of 120 ohms. As they are put in tension, their resistance increases; as they are compressed, their resistance decreases. The Wheatstone bridge provides a way to convert these changes in resistance to changes in voltage, which are easy to work with. These voltages can be conditioned, transmitted, or stored digitally.

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Figure 1 shows a Wheatstone bridge configuration.

- Four resistors are connected in an end-to-end fashion.
- The input or excitation voltage is connected to the bridge between top and bottom nodes of the circuit.
- The output is the difference between the voltage at the left node and the voltage at the right node.
- An excitation voltage is required to convert the change in resistance (in the legs of the bridge) to a change in voltage at the output of the bridge.

For the bridge shown, the output voltage is expressed as

\[ V_o = \left( \frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right) V_i \]

Figure 2: equation (1)

When building a Wheatstone bridge with strain gages, all four resistors have the same nominal value. Bridges can be built in the following configurations:

- Quarter Bridge- One strain gage and three fixed resistors
- Half Bridge- Two strain gages and two fixed resistors
- Full Bridge- Four strain gages
4.2 Quarter Bridges

Figure 3 illustrates a quarter bridge configuration. The quarter bridge has one active leg, i.e., one leg with a changing resistance. From equation (1) above we can derive an expression for the output voltage as a function of the resistance change $\Delta R$:

$$V_o = \left( \frac{R_2}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right) V_i = \frac{R}{(R - \Delta R) + R} - \frac{R}{R + R} V_i$$

$$= \left( \frac{R}{2R - \Delta R} - \frac{1}{2} \right) V_i = \left( \frac{2R - (2R - \Delta R)}{2(2R - \Delta R)} \right) V_i$$

$$= \left( \frac{\Delta R / R}{2(2 - \Delta R / R)} \right) V_i$$

$$\approx \frac{\Delta R}{4R} V_i.$$

Figure 4: equation (2)
4.3 Half and Full Bridges

Figure 5 and Figure 6 show half-bridge and full-bridge configurations respectively.

- **Half bridge**: two active legs, one in tension and one in compression. These legs are adjacent legs in the bridge.
- **Full bridge**: four active legs, two in tension and two in compression. The gages in tension are on opposite legs of the bridge.

Using equation 1 and Figure 1 as a guide, derive expressions for the output voltage of the half-bridge and full-bridge circuits.

**Thought Question**

A half bridge could be made with two gages in tension on opposite legs. When would this be useful? What would be the main problem with doing this?

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**Figure 5: Half Bridge Configuration**
4.4 Part 2: Calibration of the Strain-gaged Cantilever Beam

Your TA will provide an aluminum beam instrumented with strain gages.

1. Clamp the beam securely to the edge of the lab table with the gaged portion of the beam cantilevered beyond the edge of the bench.

2. Using the junction box provided, connect a single strain gage in a quarter bridge configuration. (Refer to Figure 3 as a guide.)

3. Connect the junction box output cable to Channel 2 of the SCXI-1520 strain gage module. The SCXI-1520 provides an excitation voltage to the bridge and amplifies the output voltage from the bridge to ranges that are easily observable or acceptable for data acquisition. The excitation voltage $V_i$ is 5 V and the gain is 500. When connected to the signal conditioning board, the amplified bridge output can be read by the data acquisition software.

4. Open the “Lab4” VI that you created in Temperature Measurement and First-Order Dynamic Response.

5. Hang weights from the end of the beam.

6. Record the voltage measured with LabVIEW in your lab book.

7. Repeat steps 5 and 6 for several different weights.

8. In Excel, plot the voltage (input) versus weight (output).

9. Find the best fit linear relationship for the data. The resulting equation can be used to calibrate the voltage output of the strain gages.

10. Apply several loads not used for calibration to test the validity of the linear curve fit.

4.5 Part 3: Comparison of Theoretical Strain to Measured Strain

In this step, we will compare a theoretically based estimate of strain for a given load to that which was measured earlier. First we will determine the strain corresponding to the voltage measurements of step 2.

1. Measure the excitation voltage for bridge circuit, $V_i$. 

Figure 6: Full Bridge Configuration

http://cnx.org/content/m13779/1.1/
The circuit analyses from step one showed that the bridge output $V_o$ is related to the excitation voltage by the following relationship

$$V_o = K \frac{\Delta R}{R} V_i$$

**Figure 7:** equation (2)

where $K$ equals $1/4$ for a quarter bridge, $1/2$ for a half bridge, and $1$ for a full bridge. The voltage measured in LabVIEW is related to the bridge output by

$$V_{\text{display}} = K_{\text{amp}} V_o + V_{\text{offset}}$$

**Figure 8:** equation (3)

where $K_{\text{amp}}$ is the gain of the signal conditioning board. Because the bridge resistances are not balanced exactly and the weight of the beam itself produces some strain, you will observe a nonzero output voltage when there is no load applied.

2. Measure the output voltage with no load. Call this voltage $V_{\text{offset}}$.

3. To determine the strain induced by the applied loads, measure the changes in the display voltages relative to this offset voltage ($V_{\text{display}}-V_{\text{offset}}$).

For a strain gage, the gage factor is defined as

$$GF = \frac{1}{\varepsilon} \frac{\Delta R}{R}$$

**Figure 9:** equation (4)

where $\varepsilon$ is the strain experienced by the gage. The gages used in this lab have a gage factor of $2.12 \pm 0.8\%$.

4. Derive an expression for the strain in the beam using equations (2) through (4). Your empirical strain estimate should be in the range of 0 to 2000 microstrains.

A theoretical estimate of the strain can be obtained by drawing on topics from CEE 203. The stress on the surface of a beam in bending is given by

$$\sigma = \frac{M y}{I}$$

**Figure 10:** equation (5)
where \( M \) is the applied moment at the location of interest, \( y \) is the distance from the neutral axis (in this case, the half height of the beam cross section), \( I \) is the area moment of inertia of the cross section with respect to the neutral axis. Recall that for a rectangular cross section,

\[
I = \frac{1}{12}bh^3
\]

**Figure 11:** equation (6)

Recall also that stress and strain are related by Young’s modulus:

\[
\sigma = E \varepsilon
\]

**Figure 12:** equation (7)

For aluminum, \( E = 10.4 \times 10^6 \) psi.

5. Using equations (5) through (7), estimate the strain where the gages are bonded to the beam. How does the theoretically obtained strain compare to the value determined from measurements? If they are different, what are some possible reasons?

**4.6 Part 4: Experiments with Half and Full Bridges**

Repeat parts 2 and 3 for the half bridge and the full bridge configurations.

**4.7 Part 5: Measurement of an Unknown Load**

Based upon the calibration determined in step 2, use your beam to determine the weight of an arbitrary object. You may use the quarter bridge, the half bridge, or full bridge configuration for this test. Measure the actual weight using a precision scale. How does the weight determined with your beam compare to the object’s true weight? How certain is your measurement? What are some possible sources of uncertainty?

**4.8 Part 6: Sensitivity to Extraneous Loads and Temperature Variations**

Strain gages are often used as transducers to measure force in a structure or mechanical device. A perfect sensor is sensitive only to changes in the variable of interest and is completely insensitive to changes in extraneous parameters. (Such a sensor is impossible to create.) In practice, every effort should be made to minimize sensitivity to extraneous variables. In the case of the cantilever-beam scale, the objective is to measure the weight of objects or alternatively, to measure forces in the vertical plane. As such, a good design will be much less sensitive to loads in other directions (axial, lateral, torsional, etc.). Also a good design will not be sensitive to strains induced by other means, such as thermal expansion.

Your TA will supervise in the following two demonstrations.

**4.8.1 6.1 Lateral Sensitivity**

Use the instrumented beam of square cross section provided by your TA.

1. Clamp the beam in a cantilever fashion so that it will measure vertically applied loads.
2. Using your hand, gently apply lateral and axial loads to the beam. Do your best to not apply vertical bending loads to the beam.

3. Compare the sensitivity of the quarter bridge to that of either the half bridge or full bridge to these extraneous loads. Which configuration seems to work the best overall? Explain why this is so. Which extraneous loads are "rejected" by which configurations? Explain why. (Note: The steps of this paragraph are more easily said than done. The point is that the quarter bridge is more sensitive to off-axis loads than either the half bridge or full bridge. If you have difficulty demonstrating this, don’t stress out.)

4.8.2 6.2 Temperature Sensitivity

Using a quarter bridge configuration, apply a load to the beam and measure the voltage output.

Apply heat uniformly to the beam (140 F max) near the gages and observe any changes in the voltage output. Do not apply heat to the gages directly! Why does the voltage (which corresponds to the measured strain) change? Repeat the procedure using a full-bridge configuration. Compare the sensitivity of the quarter bridge and full bridge configurations to temperature variations. How would you expect the half bridge to behave?

4.9 Part 7: Dynamic Characteristics

Up to this part of the lab, you have examined the static characteristics of the strain-gage bridge. Now you will modify the VI that you developed for Temperature Measurement and First-Order Dynamic Response to measure the dynamic characteristics of a signal from the strain-gage bridge. You will excite the dynamics of the cantilever beam by plucking it. You will use your VI to display the response of the vibrating beam.

4.9.1 7.1 Differential Model

The first mode of vibration of the cantilever beam can be modeled using a simple mass-spring-damper model. This model results in a second-order differential equation that describes the dynamics of the system. You will observe that the response of the strain-gages to an initial deflection is a damped sinusoid. This is the expected response for a second-order system. Using acquired data; you will compute and display values for the damping ratio and the natural frequency of the first vibrational mode of the beam.

4.9.2 7.2 Damping Ratio and Damped Natural Frequency

Figure 5 shows the response of a second-order system to an initial condition. This response plot will be used to define the damping ratio and the natural frequency for this system. From the amplitudes $x_1$ and $x_n$, the damping ratio can be calculated using the following expression:

$$
\zeta = \frac{\frac{1}{x_1} \ln \left( \frac{x_1}{x_n} \right)}{\sqrt{4\pi^2 + \left[ \frac{1}{x_1} \ln \left( \frac{x_1}{x_n} \right) \right]^2}}
$$

Figure 13: equation (8)

The damped natural frequency can be determined by measuring the period of the damped oscillations. An accurate measurement of the period $T$ is obtained by considering several periods:

http://cnx.org/content/m13779/1.1/
\[ T = \frac{t_2 - t_1}{n - 1} \]

**Figure 14:** equation (9)

\[ \omega_d = \frac{2\pi}{T} \]

**Figure 15:** equation (10)

Once and are known, the natural frequency of oscillation can be calculated by

\[ \omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} \]

**Figure 16:** equation (11)
4.9.3 Programming Exercise

You will modify your VI to automate the calculation of and for the cantilever beam.

1. Your TA will direct you to a subVI named “Damping Ratio” that is has been started for you.
2. To place the subVI on your block diagram, open the functions palette and select Select a VI...
3. Connect the Data wire from the DAQ Assistant to the Measured Data input of the Damping Ratio subVI.
4. Open the block diagram by double-clicking the subVI icon. (The block diagram is shown in Figure 18.)
5. You will need to modify the block diagram for the VI to work. Refer to equations 8-11 as you design a block diagram that will calculate the Damping Ratio of your signal.
6. You will notice that the necessary inputs for the subVI are placed along the left of the block diagram. The outputs are placed along the right of the block diagram. The Peak Detector VI is used to locate the peaks (or valleys) of the signal. The locations of the peaks are given as indexed data points. For example, if the first peak occurs in between the sixth and seventh data points, the location will be 6.5. The time at which this peak occurs can be calculated using the sample rate.

7. For more information about the Peak Detector or other VIs, press Ctrl+H to activate the context help window.

(Hint: you may want to use the index array function to pull values such as x1, xn, t1 and tn out of the amplitude and location arrays.)

This exercise is meant to give you a feel for the possibilities for data acquisition, processing, and display using computer software tools. Be sure to save a copy of your data for plotting. You will need it for your writing assignment.

4.10 Part 8: Design of a Force Transducer

For your course project, you will design a force transducer for measuring the thrust of a model rocket motor. This exercise is intended to help you initiate the design process. You may assume that the maximum thrust that will be sensed by your transducer is 5 lb.

4.10.1 Cantilever-Beam Force Transducer

Assume that you will use a cantilever-beam configuration similar to what was used in this lab. A full strain-gage bridge will be used. The width of two gages side-by-side is 1/2 inch. A good design will result in 2000 microstrains under the maximum load and will have a natural frequency above 100 Hz. Meeting both of these objectives might not be possible with a cantilever-beam design. You are to determine the thickness of the cross section where the gages are to be mounted and the distance from where the motor is mounted to the center of the gages (i.e., the length of the cantilever).
4.10.2 Spreadsheet Analysis

Develop a spreadsheet to perform your calculations. A spreadsheet will simplify the evaluation of different designs. This analysis will be a good starting point for the analysis of your project. Note: Your project will involve different loads and different dimensions than the transducer used in this lab. The calculations, however, are the same.

5 Lab Report

For this lab you will write the Results and Discussion of Results sections of a full report. As you perform the lab, think about what data should be saved or recorded for presentation and why these data are important. For some of your data, tabulation is sufficient (e.g., calculated strain vs. measured strain). Other data should be recorded using LabVIEW (e.g., dynamic response of strain when the beam is plucked). Make sure that you include your thoughts about the results you obtained and why they are important. Discuss their agreement with your expectations.